Tsunami in DERIVE and TI-92 (Studding Solitons)

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Hard&Soft: TI-92, DfW & DfD v.4, Cyclon99, DpGraph, Hotkeys, Scancode, FastPlot

Abstract

Tsunami is a big solitary wave i.e. soliton. Soliton is widespread natural non-linear phenomenon described by nonlinear partial differential equations. Its properties can be clearly illustrated in DERIVE and TI-92. The workshop «Studding Solitons» for graduate students of the Moscow Pedagogical State University is presented. Tsunami movie (big soliton) and its implementation with our package FastPlot that integrates DERIVE with the 3D surfaces real time animator Cyclon99 and DPGraph is described.

1. Solitons and its Properties

The forthcoming age seems to be nonlinear - nonlinear physics and mathematics are coming. Soliton is a widespread natural non-linear phenomenon. It can be observed in the ocean as a big solitary wave tsunami, as a nerve impulse, or plasma impulse etc. Solutions of a large class of nonlinear partial differential equations with dispersive terms are solitons. The soliton stability phenomenon is qualitatively explained as the mutual nonlinearity and dispersion compensation.

First solitary wave phenomenon was observed in the water channel and described by the British marine engineer John Scott Rassel in 1834 and the appropriate equation derived by Karteweg and de Vries in 1895 [1]. At the beginning of this century the idea of the connection between solitary waves and particles appeared. It was induced by the fact that such waves preserve their main properties (shape and velocity) after interaction. That is why Zabusky N.S. and Kruskal M.D. (1965) abbreviated the name of these phenomenon to «Soliton» using the analogy with proton and neutron. This idea of particles and solitons connection is actively developing in the Elementary Particles Physics now.

Solitary waves at the shallow water are described by the Korteweg de Vries differential equation in partial derivatives:

$$u_t + u \cdot u_x + b \cdot u_{xxx} = 0; \quad u = u(x;t) \tag{1}$$

Deep water solitons, magneto - sonic, and ionic -sonic solitons - by nonlinear Schroedinger equation [2]

$$\Psi_z + D \cdot \Psi_{zzz} + G \cdot |\Psi| \cdot \Psi = 0, \quad \Psi = \Psi(z,t) \tag{2}$$

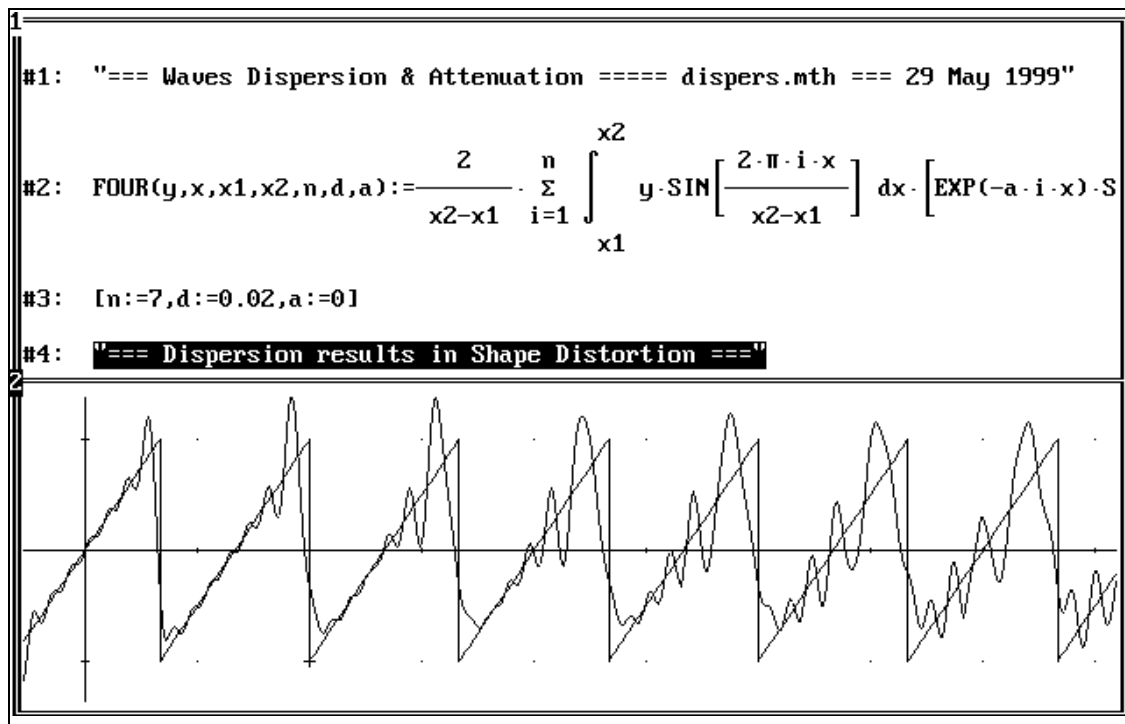with the non-linearity and dispersion factors G and D respectively.

Solitons in superconductors and ferromagnetics - by Sine - Gordon equation [3]

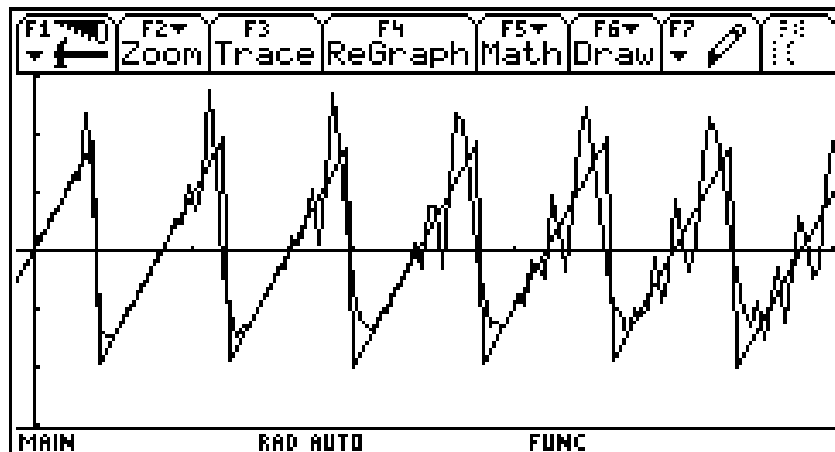$$\Psi_{zz} - \Psi_{tt} = \sin\Psi, \qquad \Psi = \Psi(z,t) \tag{3}$$

All these equations are one-dimensional. Multidimensional solitons are not well studied yet.

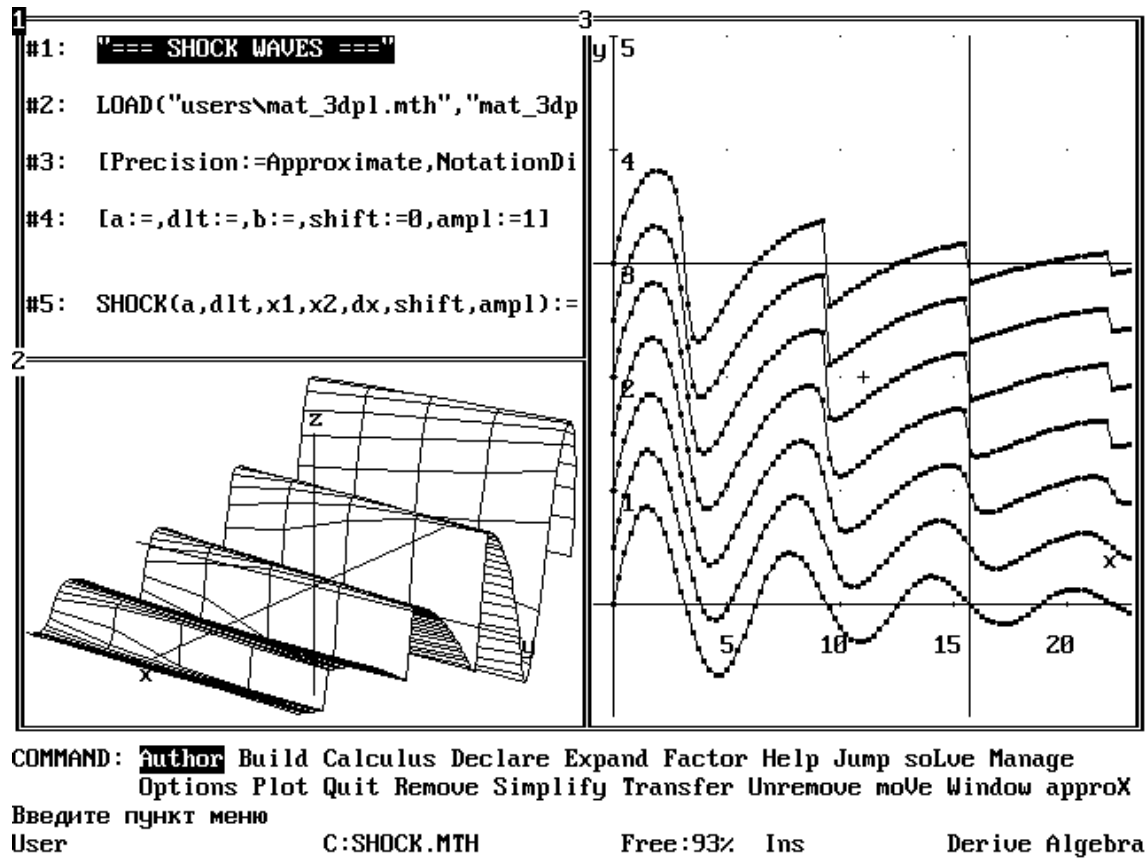## 2. Solitons Workshop. Part 1: Computer Simulation

In the computer simulation part of the Solitons workshop first students derive and observe the distortion of a saw tooth wave in dispersive media in DERIVE

or in TI-92/89 graphic calculator.

Then they observe the sine - wave transformation to a shock wave in nonlinear media [4]. The appropriate nonlinear equation has no symbolic solution. So, it is solved numerically and the result can be presented in 2D (right window), using nonlinearity factor as a parameter, or in 3D (window 2) forms. The 3D form can be rotated in Acrospin.
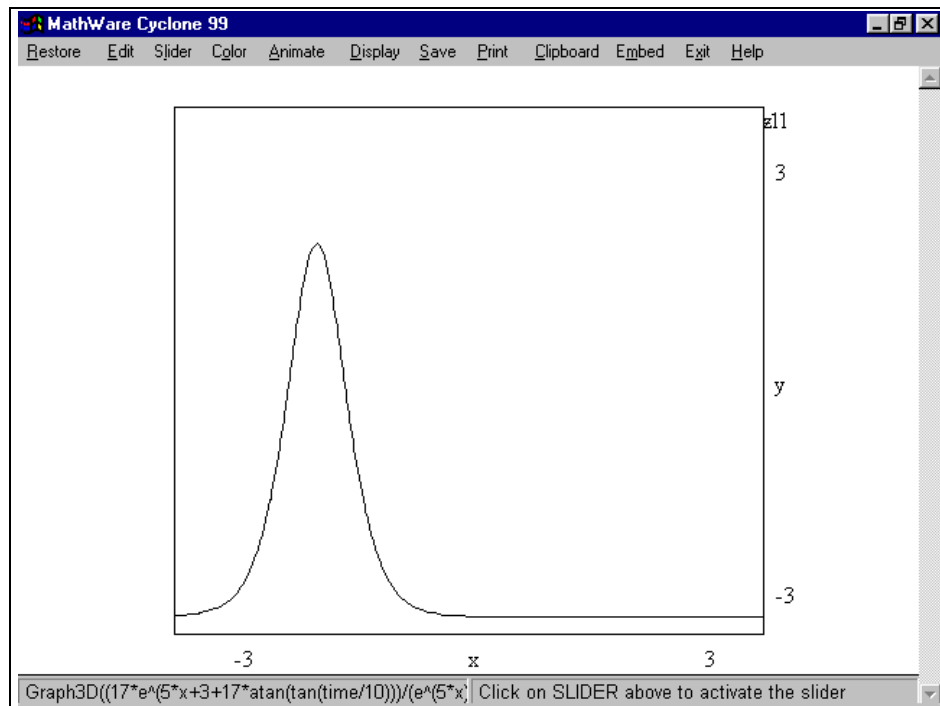

At the third step they observe the result of nonlinearity and dispersion compensation that results in wave shape conservation. Most of equations corresponding solitons can be solved only numerically, but there are still some ones with symbolic solutions. The solution of Korteweg - de -Vries equation for one soliton with the velocity a and initial (t=0) position s is:

$$u = a^2 \frac{f}{(1+f)^2} \; ; \; f = exp(-a(x-s) + a^3 t); \; u = \frac{e^{x+t}}{(e^x + e^t)^2} \qquad (4)$$

It can be demonstrated as a Movie in Cyclon99 or DPGraph.

The screenshot of Cyclon99 with the soliton moving from left to right is below. It is an example of a univariate function movie.
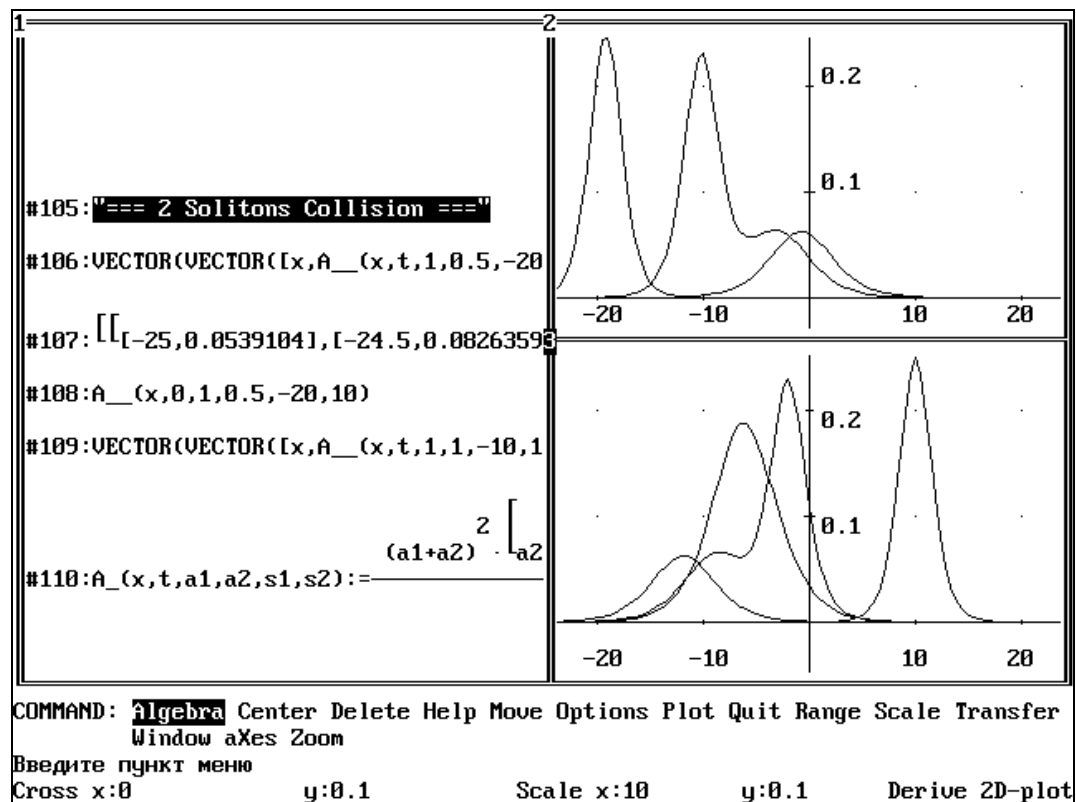
```
MathWare Cyclone 99                                    _ | 8 | X
Restore  Edit  Slider  Color  Animate  Display  Save  Print  Clipboard  Embed  Exit  Help
```

```
Graph3D((17*e^(5*x+3+17*atan(tan(time/10)))/(e^(5*x  Click on SLIDER above to activate the slider
```

The analogous but not so impressive movie can be done in DERIVE for DOS with the help
of Scancode keystroke simulator (http://members.aol.com/bretjohn). One have to prepare a
vector of curves for different time and press Alt-F9. The simulator highlights the next
element of the vector, switches to the 2D window, deletes previous plot, plots the
highlighted one and switches back to the vector etc. If the computer is rather fast for the
equation plotted, it seems that the curve moves and/or changes.

The Korteweg - de - Vries equation has not only one, but multi-solitons solutions also. The
Two - Solitons Solution is:

Fehler!   *(5)*

where, $f_i = exp(-a_i (x-s_i)+a_i{}^3t)$, $a_{21} = \dfrac{a_2 - a_1}{a_2 + a_1}$

It is wonderful that two solitons at the screenshot below are transmitted through each other
without scattering as nothing had happened (except phase shift). The big one has a greater
positive velocity, catches up the small one (window 2), and outstrips it. Note that the height
of the sum of two solitons is less than the height of the biggest one, but it is thicker than
both interacting solitons.

```
1═══════════════════════════2═══════════════════════
                                          │ 0.2
                                          │
                                          │ 0.1
 #105:"=== 2 Solitons Collision ==="      │
                                          │
 #106:VECTOR(VECTOR([x,A__(x,t,1,0.5,-20  │ ─────────────────────
                                       -20  -10    10    20
 #107:[[[-25,0.0539104],[-24.5,0.08263598
                                          │
 #108:A__(x,0,1,0.5,-20,10)               │ ╱╲        ╱╲    │ 0.2
                                          │╱  ╲      ╱  ╲   │
 #109:VECTOR(VECTOR([x,A__(x,t,1,1,-10,1  │    ╲    ╱    ╲  │
                                          │     ╲  ╱      ╲ │ 0.1
                            2  ┌          │      ╲╱        ╲│
                     (a1+a2)  │a2         │ ─────────────────────
 #110:A_(x,t,a1,a2,s1,s2):=────────        -20  -10    10    20

COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Введите пункт меню
Cross x:0              y:0.1          Scale x:10       y:0.1    Derive 2D-plot
```
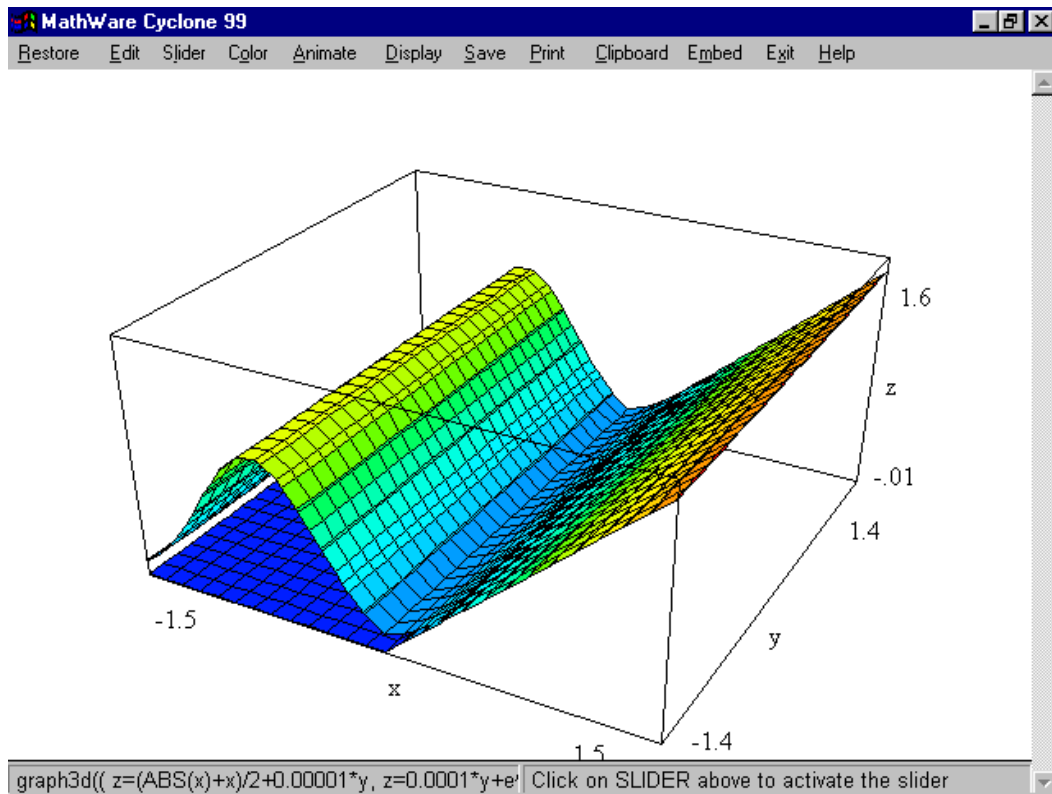
Using the solution for one soliton (4) students calculate solitons position, velocity, width, height, mass, energy and apply it to the problem of the tsunami at the shore.

The screenshot of Cyclon99 (http://www.davidparker.com) with the tsunami moving to the shore is below. It is a bivariate function movie.
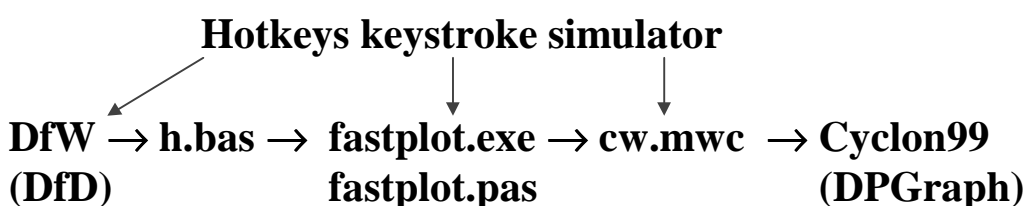
Most of the described tasks are optional in the workshop and the number of tasks and its difficulty is adjusted to every student individually.

The second part of the workshop simulates solitons in a nonlinear two-meter long line of linear and nonlinear capacities and inductances at 2 MHz [5].

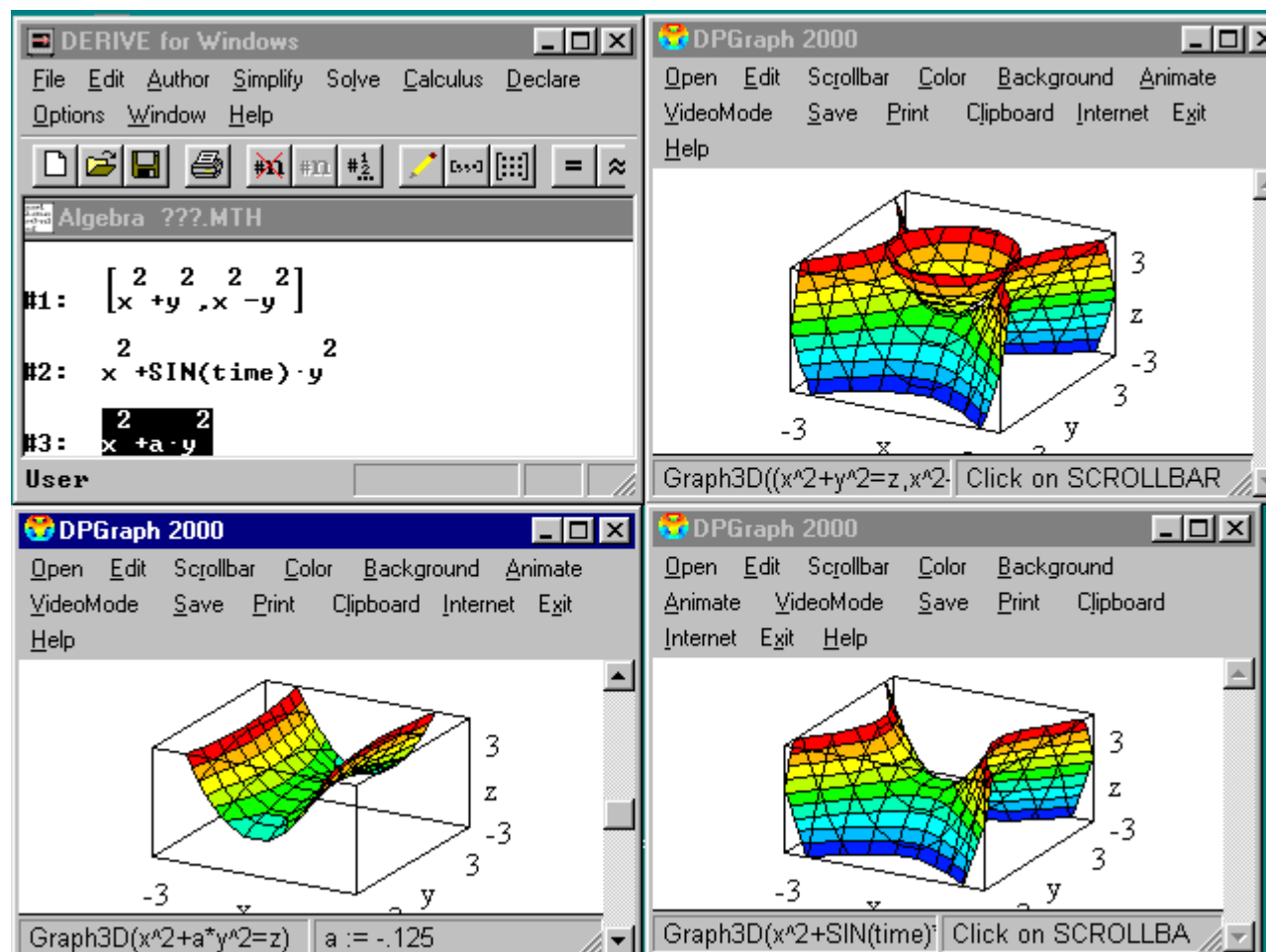# 3. One keystroke Movies in DERIVE + Cyclon99/DPGraph
(FASTPLOT)

There are no all - purpose programs and one has to switch between different ones and make format conversion. Usually it needs more than ten keystrokes and distracts from the main problem solution. Programs can be integrated with the Macro of one of the programs or with one of keystroke simulators that presses keys instead of us. Postcardware simulator Hotkeys for Windows 95 and DOS   (http://members.xoom.com/PostcardWare) was selected for Windows and DOS versions of DERIVE.

The structure of our FastPlot utility is below:

Hotkeys keystroke simulator

DfW → **h.bas** → **fastplot.exe** → **cw.mwc** → **Cyclon99**
(DfD)   **fastplot.pas**   **(DPGraph)**

It supports one keystroke real time 3D & 2D plots directly from DERIVE. Real time rotation, scaling, and parameter change of the plotted surfaces are possible.

The below example shows 3 expressions and its plots: two surfaces $z=x^2+y^2$ and $z=x^2-y^2$ (right-top window), one surface $z=x^2-\sin(time)\cdot y^2$ with a time dependent parameter (a movie), and a surface $x^2-a\cdot y^2$ with a parameter controlled by a scrollbar (bottom left window). The parameter value $a=-.125$ is at the bottom message line.

The utility presented makes 3D surfaces plots and movies easy and impressive.

The used keystroke simulators can be also used for automating repetitive tasks in DERIVE, making demonstrations and initializing DERIVE state i.e. to load files, open/close windows, simplify expressions, input some text etc.

References

1. Korteweg D.J., de Vries G. On the change of form of long waves advancing in rectangular channel and new type of long stationary waves. / Phil. Mag. 1895, **39**, p.442-443.
2. Whitham G.B. Linear and nonlinear waves. John Wiley & Sons, 1974
3. Rubinstein J. Sine - Gordon equation. / J. Math. Phys., 1970, **11**, p.258-268.
4. Migulin V.V. et. al. Oscillation Theory Foundations. Moscow, «Nauika, Fizmatlit», 1978, 392 p. (Russian)
5. Fydorova J.V., Iljin V.A., Maslenikov N.M., Biryukov S.V. Simulation of the Waves Propagation in the Nonlinear Dispersive Media / Physical Education in High School, Moscow, N19, 2000 (Russian, to be published)