TI-92 programs in Maths lessons

At first sight one sees that the TI-92 has a large number of built-in commands. The application number 7 : Program Editor does not seem to be very necessary. With time there are many good reasons for teachers and pupils to use the program-editor. I was compelled by my gifted pupils to use the program-editor. They made programs to do easy homework and tests. The untalented pupils used these programs without understanding the mathematical background. So I began to instruct all my pupils in learning programming with the TI-92. Today in my opinion there are five different possibilties to use TI-92 programs in lessons:

1) Transformation of small mathematical problems to small programs.

Even if pupils are able to solve a problem without the TI-92, it is not easy for them to make a program to solve this problem. The finding of an algorithm or only special cases (for example the division by Zero) or syntax errors are often insuperable difficulties. Programs of this type should be small. In most instances it is not necessary to use complex commands such as the command to create a dialog box. The advantage of small programs is that an example of a test can be to create a such small program.

Now look at the following example. The parametric equation of a line (vector-equation) should be transformed into a linear equation.

$$g: \vec{X} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

The coefficients a und b of the linear equation ax + by = c are the

coordinates of the vector $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ which is normal to the vector $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$. Therefore

the linear equation has the form 4x + 5y = c. The constant term c can be

calculated by substitute x and y by the coordinates of the Point $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ of g:

A simple possibility of solving this problem in a program is to use the coordinates of the point and the vector as parameters of the call of the program:

<div align="center">program(3,-2,5,-4)</div>

But pupils will use this program for the application at the homescreen and therefore it is more comfortable to use the whole equation g as parameter:

<div align="center">program(g)</div>

Now the problem arises to calculate the coordinates of the point $P = \begin{pmatrix} px \\ py \end{pmatrix}$ and

the vector $\vec{r} = \begin{pmatrix} rx \\ ry \end{pmatrix}$ of the equation $\vec{x} = P + t \cdot \vec{r}$ and not only to see the values.

A deeper understanding of the importance of the parameter t in the equation g is necessary. Also the command:**right**, the **with operator** and the **grip to the coordinates of a vector (matrix)** must be used.
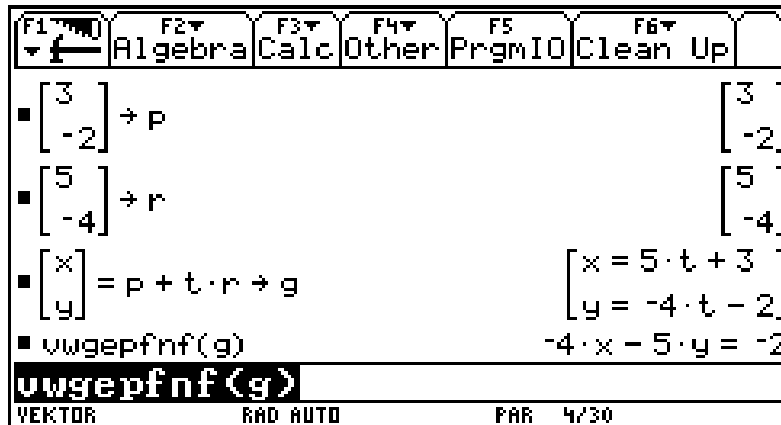Here you can see the code of this program.

```
F1▾  F2▾    F3▾ F4▾   F5        F6▾
▾⌐Control I/O Var Find... Mode
:vwgepfnf(g)
:Func
:Local rx,ry,px,py
:(right(g[1,1])|t=1)-(right(g[1,1])|t=0)
 →rx
:(right(g[2,1])|t=1)-(right(g[2,1])|t=0)
 →ry
:right(g[1,1])|t=0→px
:right(g[2,1])|t=0→py
:ry*x-rx*y=ry*px-rx*py→g
:EndFunc

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```

It is a small program for a small problem. But it is not trivial. It is a nice example for a Test. Such programs are a possibility of teaching all pupils to program with the TI-92.
The following picture shows you an example how to use this program.

```
F1 ▼   F2▼     F3▼  F4▼    F5     F6▼
▼ ┌── Algebra Calc Other PrgmIO Clean Up
```

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix} \rightarrow P \qquad\qquad\qquad \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -4 \end{bmatrix} \rightarrow r \qquad\qquad\qquad \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = p + t \cdot r \rightarrow g \qquad \begin{bmatrix} x = 5 \cdot t + 3 \\ y = -4 \cdot t - 2 \end{bmatrix}$$

$$\blacksquare \; \text{vwgepfnf}(g) \qquad\qquad -4 \cdot x - 5 \cdot y = -2$$

vwgepfnf(g)

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```

2. Working with modules.

Analytic Geometry is a big part of the Austrian curriculum. But there are only four built in commands for analytic geometry in the TI-92: norm, dotP, crossP and unitV. Therefore I developed many programs which allow to split up complex problems of analytic geometry into simple modules.
Such modules are for instance: point of intersection of two lines
point of intersection of a line and a plane
intersection of two planes
intersection of a circle and a line
equation of a tangent line to a circle
equation of the perpendicular bisector
equation of the angle bisector and so on

These modules are not used as black boxes by the pupils. During the lessons pupils learn to program these modules step by step as shown in section 1.
Now let us show an example: The vertices of a triangle are given.

$A = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $B = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$ and $C = \begin{pmatrix} 5 \\ 29 \end{pmatrix}$. The point of Gergonne (The lines containing a vertex of a triangle and the point of tangency of the opposite side with the inscribed circle are concurrent) should be calculated.
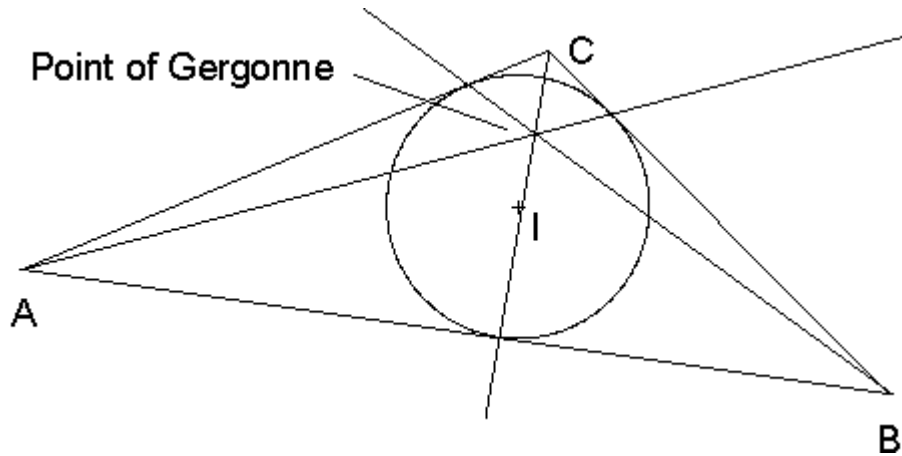
At first the coordinates of the vertices are stored.
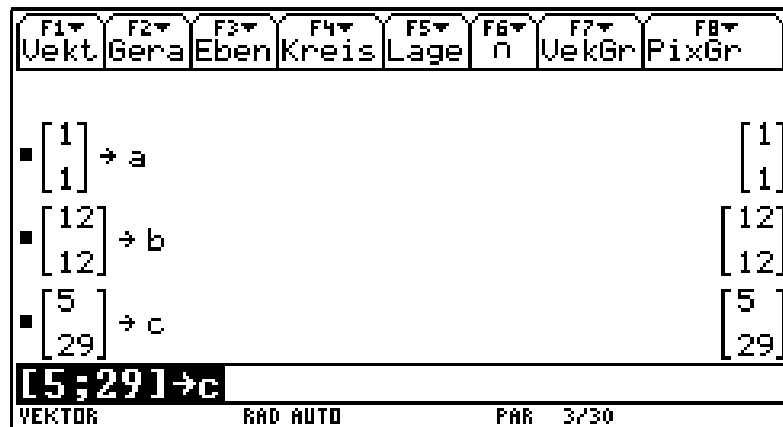
Then we calculate the angle bisector angbiα of the angle α and angbiβ of the angle β with the module winsypkt(). This module constructs the equation of the angle bisector.

Now we get the midpoint incimd of the inscribed circle as intersection point of the two angle bisectors with the module sngeger2(). This module calculates the intersection point of two lines. The radius r of the inscribed circle is the distance from the midpoint to one side of the triangle. The module abspktge() calculates the distance of a point to a line. The equation of the inscribed circle eqci is calculated with the module krmpktra()

The points of tangency tab, tac, tbc of the inscribed circle with the sides of the triangle is calculated by the module sngekr(), which calculates the intersection points of a line and a circle.

```
┌─F1▾─┬─F2▾─┬─F3▾─┬─F4▾─┬─F5▾─┬─F6▾─┬─F7▾─┬─F8▾─┐
│Vekt │Gera │Eben │Kreis│Lage │  ∩  │VekGr│PixGr│
├─────┴─────┴─────┴─────┴─────┴─────┴─────┴─────┤
│ ■ abspktge(incimd,gezwpkt(a,b)) → r     3·√2  │
│ ■ krmpktra(incimd,r) → eqci                   │
│              x² − 14·x + y² − 26·y + 200 = 0   │
│                                          ⎡10⎤  │
│ ■ sngekr(gezwpkt(a,b),eqci) → tab        ⎣10⎦  │
│                                        ⎡14/5⎤  │
│ ■ sngekr(gezwpkt(a,c),eqci) → tac      ⎣68/5⎦  │
├───────────────────────────────────────────────┤
│ SnGeKr(GeZwPkt(b,c),eqci)→tab                 │
│ VEKTOR        RAD AUTO        PAR  10/30       │
└───────────────────────────────────────────────┘
```

```
┌─F1▾─┬─F2▾─┬─F3▾─┬─F4▾─┬─F5▾─┬─F6▾─┬─F7▾─┬─F8▾─┐
│Vekt │Gera │Eben │Kreis│Lage │  ∩  │VekGr│PixGr│
├─────┴─────┴─────┴─────┴─────┴─────┴─────┴─────┤
│ ■ sngekr(gezwpkt(a,b),eqci) → tab      ⎣10⎦   │
│                                        ⎡14/5⎤  │
│ ■ sngekr(gezwpkt(a,c),eqci) → tac      ⎣68/5⎦  │
│                                        ⎡142⎤  │
│                                        ⎢───⎥  │
│                                        ⎢ 13⎥  │
│ ■ sngekr(gezwpkt(b,c),eqci) → tbc      ⎢190⎥  │
│                                        ⎢───⎥  │
│                                        ⎣ 13⎦  │
├───────────────────────────────────────────────┤
│ SnGeKr(GeZwPkt(b,c),eqci)→tbc                 │
│ VEKTOR        RAD AUTO        PAR  11/30       │
└───────────────────────────────────────────────┘
```

With the module gezwpkt() we construct the equations la, lb and lc of the lines containing a vertex and the point of tangency of the opposite side.

```
┌─F1▾─┬─F2▾─┬─F3▾─┬─F4▾─┬─F5▾─┬─F6▾─┬─F7▾─┬─F8▾─┐
│Vekt │Gera │Eben │Kreis│Lage │  ∩  │VekGr│PixGr│
├─────┴─────┴─────┴─────┴─────┴─────┴─────┴─────┤
│                                        ⎡142⎤  │
│                                        ⎢───⎥  │
│ ■ sngekr(gezwpkt(b,c),eqci) → tbc      ⎢ 13⎥  │
│                                        ⎢190⎥  │
│                                        ⎢───⎥  │
│                                        ⎣ 13⎦  │
│                                    ⎡x = 10 − 5·t ⎤│
│ ■ gezwpkt(tab,c) → lc              ⎣y = 19·t + 10⎦│
│                                   ⎡x = 14/5 − 23·t⎤│
│ ■ gezwpkt(tac,b) → lb             ⎣y = 4·t + 68/5 ⎦│
├───────────────────────────────────────────────┤
│ GeZwPkt(tac,b)→lb                             │
│ VEKTOR        RAD AUTO        PAR  13/30       │
└───────────────────────────────────────────────┘
```

```
┌F1▼─┬F2▼──┬F3▼──┬F4▼──┬F5▼─┬F6▼┬F7▼───┬F8▼──┐
│Vekt│Gera │Eben │Kreis│Lage│ ∩ │VekGr │PixGr│
├────┴─────┴─────┴─────┴────┴───┴──────┴─────┤
│ ■ gezwpkt(tab,c) → lc        ⌐x - 10   ⌐   │
│                              ⌊y = 19·t + 10⌋
│                                             │
│ ■ gezwpkt(tac,b) → lb        ⌈x = 14/5 - 23·t⌉
│                              ⌊y = 4·t + 68/5 ⌋
│                                        ⌈      142 ⌉
│                                        │x = 43·t + ── │
│                                        │          13  │
│ ■ gezwpkt(tbc,a) → la                  │          190 │
│                                        │y = 59·t + ── │
│                                        ⌊          13  ⌋
├─────────────────────────────────────────────┤
│GeZwPkt(tbc,a)→la                            │
├─────────────────────────────────────────────┤
│VEKTOR        RAD AUTO        PAR  14/30      │
└─────────────────────────────────────────────┘
```

The point of Gergonne is the result of the intersection of the lines la and lb. Calculated with the module sngeger2(). At last we test with the module pktgerad if the line lc is containing the point of Gergonne.

```
┌F1▼─┬F2▼──┬F3▼──┬F4▼──┬F5▼─┬F6▼┬F7▼───┬F8▼──┐
│Vekt│Gera │Eben │Kreis│Lage│ ∩ │VekGr │PixGr│
├────┴─────┴─────┴─────┴────┴───┴──────┴─────┤
│ ■ gezwpkt(tbc,a) → la                       │
│                                      ⌐      190 ⌐
│                              y = 59·t + ──     │
│                                             13  │
│                                        ⌈ 1300 ⌉│
│                                        │ ──── ││
│ ■ sngeger2(la,lb) → gergonne           │ 139  ││
│                                        │ 1732 ││
│                                        │ ──── ││
│                                        ⌊ 139  ⌋│
│ ■ pktgerad(gergonne,lc)                     │
│                  "Liegt auf der Geraden."   │
├─────────────────────────────────────────────┤
│PktGerad(gergonne,lc)                        │
├─────────────────────────────────────────────┤
│VEKTOR        RAD AUTO        PAR  16/30      │
└─────────────────────────────────────────────┘
```

3) Ilustration and demonstration of mathematical findings

3.1) Interactive representation of the derivative

For this program you must use the display of the TI-92 in upright position. The display is bisected. In the upper part the graph of a user defined function and the tangent line at the lower bound of the display is shown. By
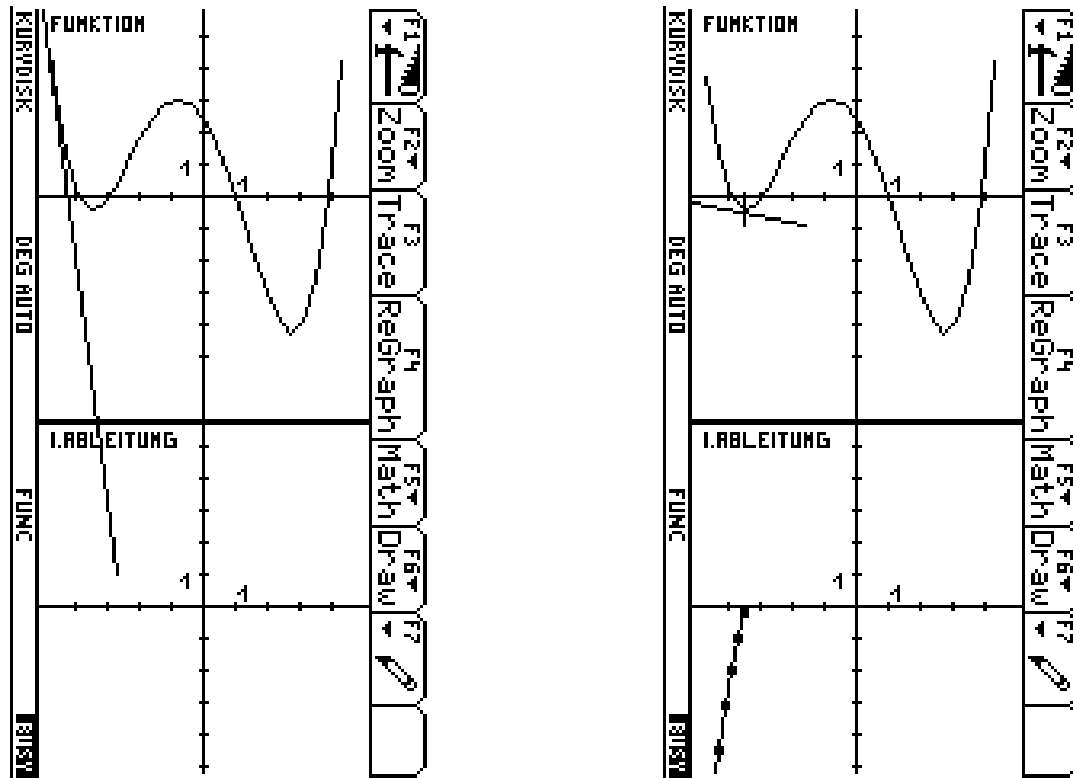
using $\odot$ the point, where the tangent line is drawn, moves to the right. In the lower part of the display the slope of the tangent line is drawn. So step by step the Graph of the first derivative of the function is generated. By using $\boxed{\text{2nd}}$ $\odot$ the lenght of stride is greater.
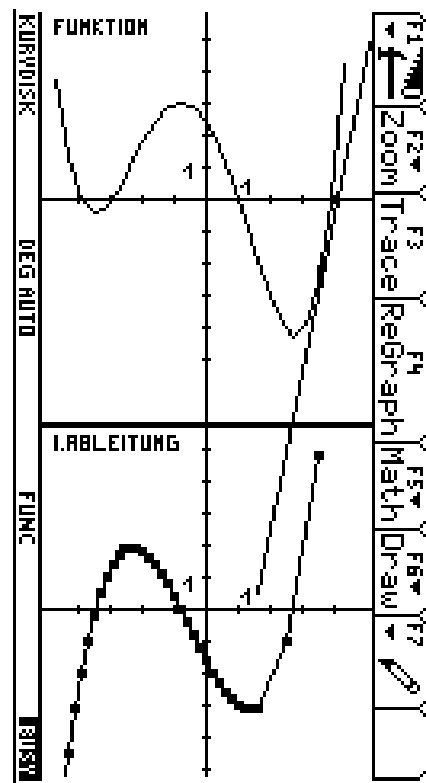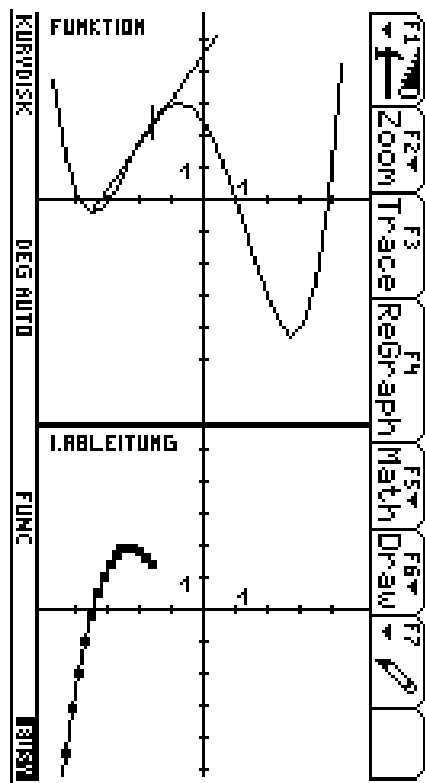
With this program pupils can explore the connection between the function and its derivative (zero, maximum point, minimum point, point of inflection...) without knowing about quotient of differences or derivative formulas. For polynomial functions they can realize the connection between the degree of the function and the degree of the derivative.

3.2) Control of the knowledge about the connection between a function and its derivative

Also for this programm you must use the display of the TI-92 in upright position. The display is bisected. By random numbers a function is selected from a library of fifty functions and its graph is displayed in the upper part

of the display. In the lower part of the display the graphs of three different functions are drawn. The pupils should see which of the graphs is the graph of the derivative of the function above. The program calls pupils to answer and control their answers.
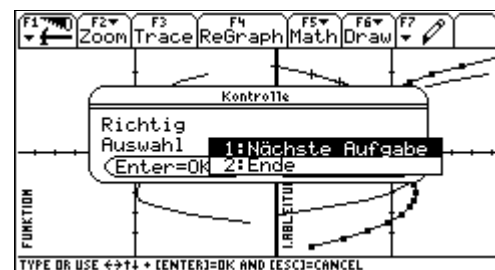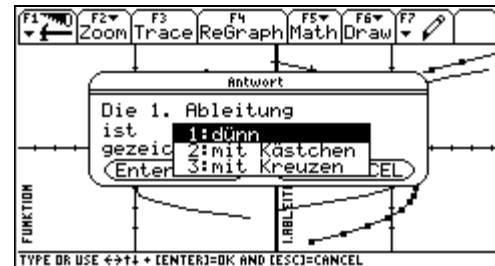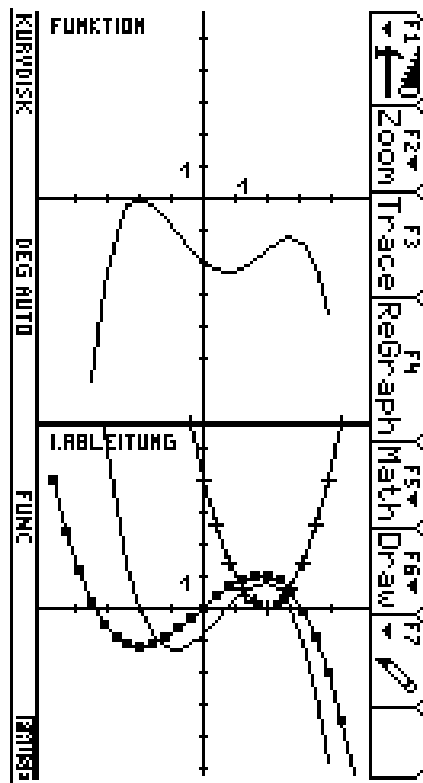
3.3) Connection between function, first derivative and second derivative

For this program you must use the display of the the TI-92 in upright position. The display is divided into three parts. At the top the graph of a user defined function is drawn. In the middle part the first derivative is drawn and in the lowest part the second derivative is displayed.

To show the connection between the maxima (minima) points of the function and the zeros of the first derivative and the points of inflection of the function and the zeros of the second derivative dotted lines are drawn. Such a picture is often seen in books about calculus. But the program gives us the possibility of showing the connection for many different functions. Pupils can test their knowledge by different functions. During the lesson many pupils get the chance to try to interpret such a picture.
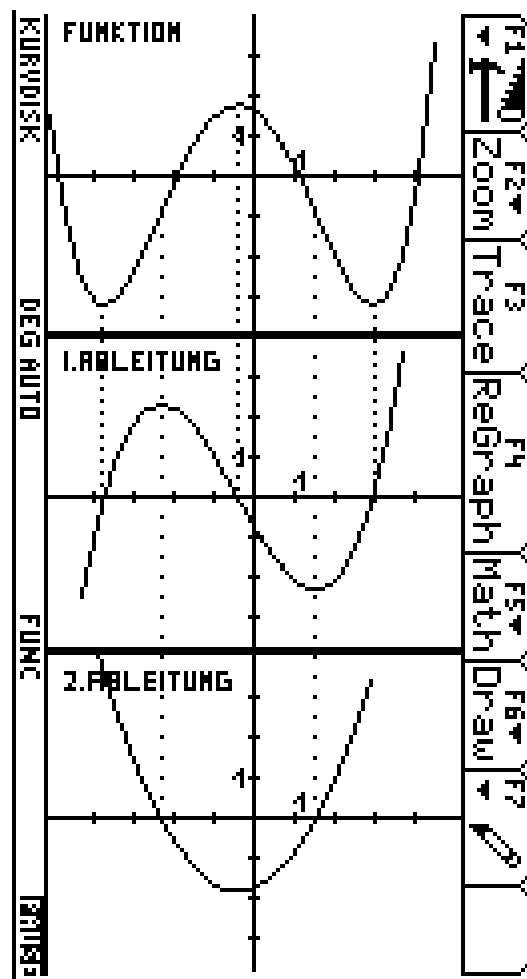
3.4) Value, slope and curvatur of a function

The display is bisected. On the right side there are drawn the graphs of two polynomials of third degree of the form $f(x) = x^3 + ax^2 + bx + c$. Therefore they are determined by the value of function (Funktionswert), the value of first derivative (slope = Anstieg) and the value of the second derivative (Krümmung = Curvature). The function with the thick graph is generated by random numbers. In the left part of the display there are buttons with which one can change the values of function, slope and curvature of the function with the thin graph.
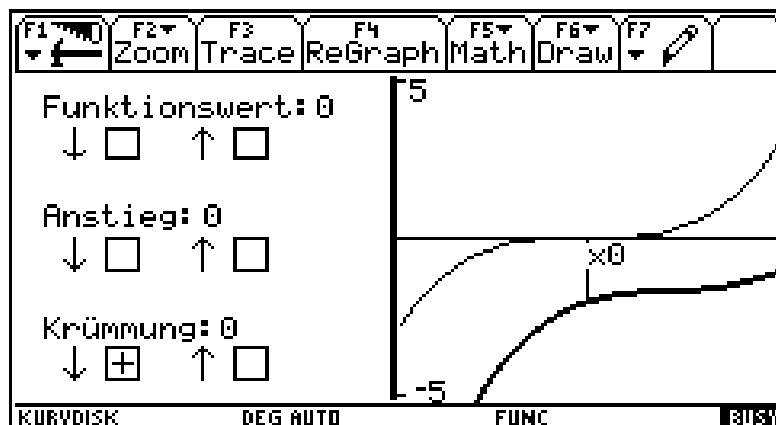
Pupils should vary the value of function, slope and curvature as long as the two graphs are identical with each user.
At first one should reach the equal value of function at xo.
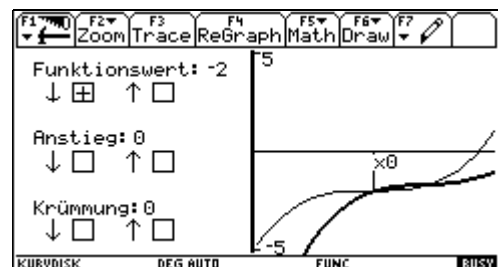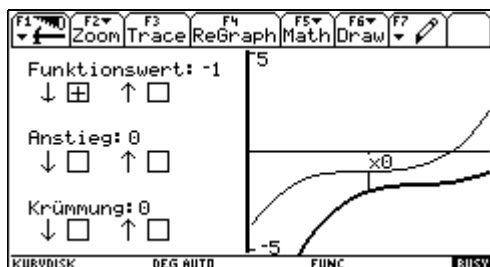
Then one should reach the equal slope at x0.
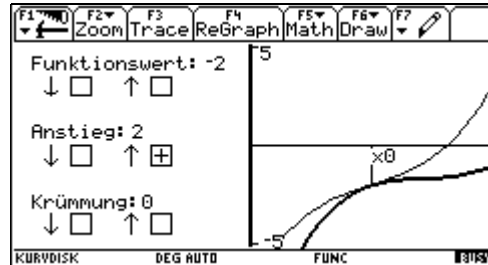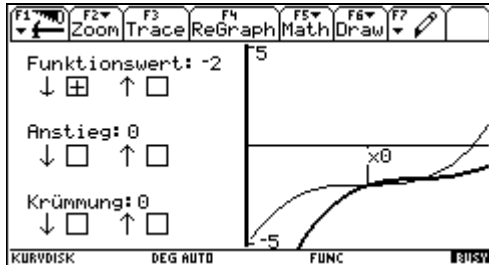
At last one should reach equal curvature at x0.
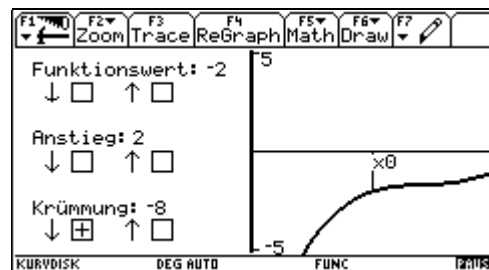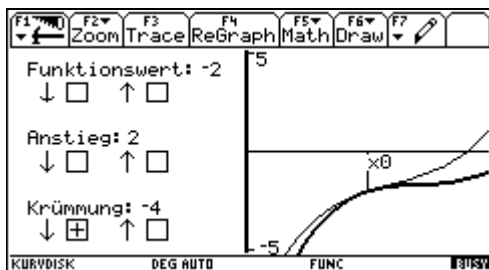
The program counts the tries and shows the minimal number of tries.
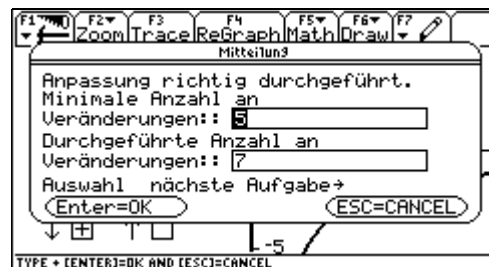
This program is good exercise for pupils to distinguish between the value of function, slope and curvature. Also it is a good training to see the sign of slope and curvature.

3.5) Fundamental Theorem of Calculus

The first program of this section should show the properties of the definite integral of a function f taken from fixed lower bound a to variable upper bound x. For this program you must use the display of the TI-92 again in upright position. The display is bisected. In the lower part the graph of a user defined function is drawn. By using $\odot$ beginning at the lower end of the displayed interval step by step the area between the function and the x axis is shaded and the value of this area ($\int_{a}^{x} f$) is drawn in the upper part of the display as function of x. So pupils can study the properties of the definite integral without knowing anything about limit or Riemann sums.
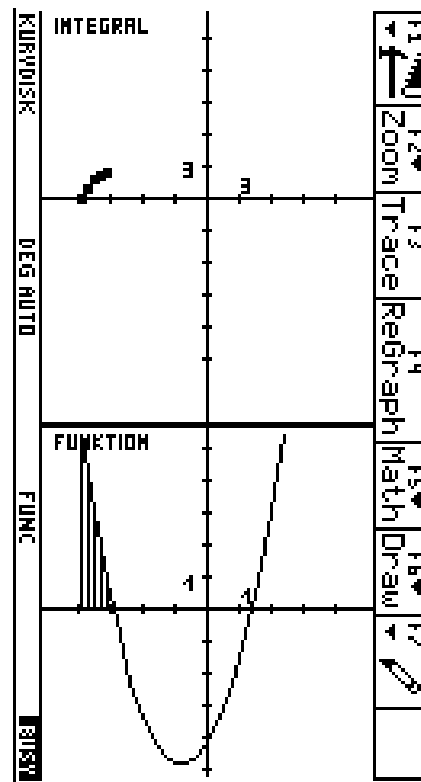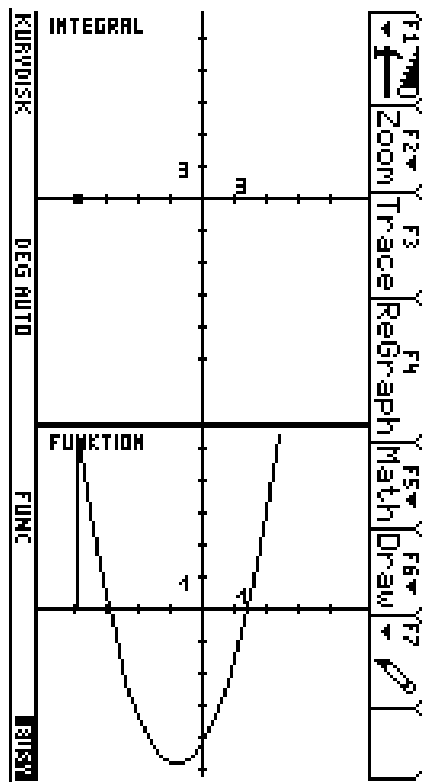
If pupils know the properties of derivative very well , they will see Fundamental Theorem of Calulus.
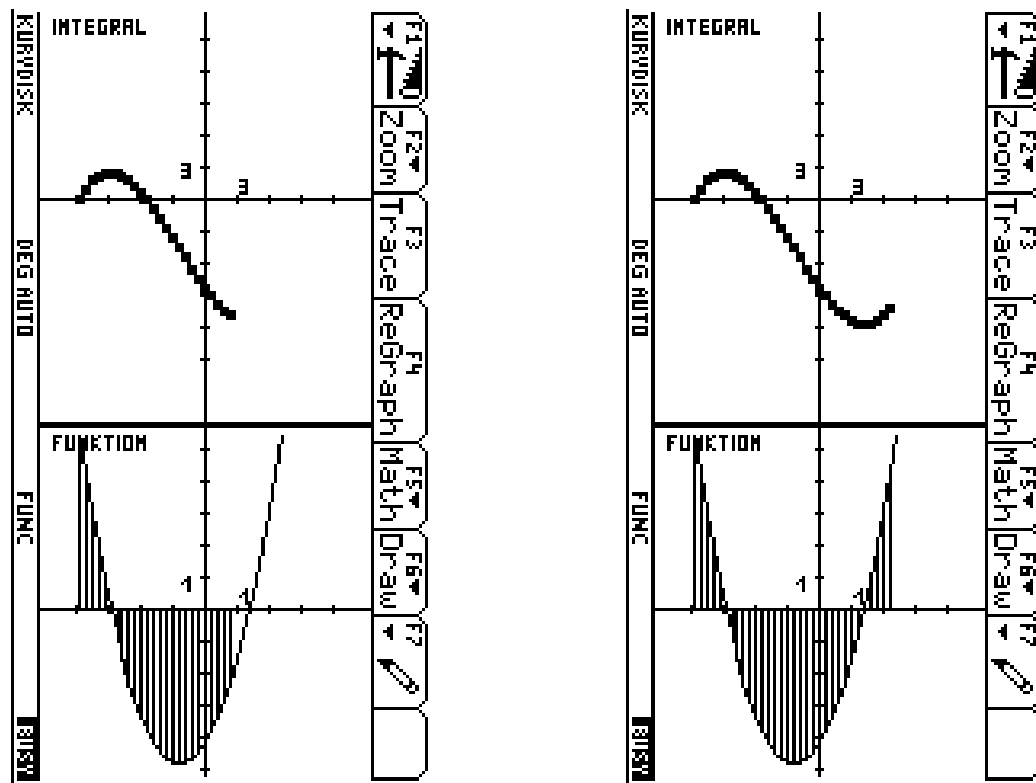
Also the following interpretation is very informative. A zero of the function causes a change of sign of the area and therefore the integral function $\int\limits_{a}^{x} f$ has a maximum or minimum point at this x-value. A maximum or minimum point of the function causes a decrease of the growing (or reducing) of the integral function $\int\limits_{a}^{x} f$ and there the integral function $\int\limits_{a}^{x} f$ has at this x-value a point of inflection.

The next program shows a possibility of proving the Fundametal Theorem of Calculus in a very instructive way.
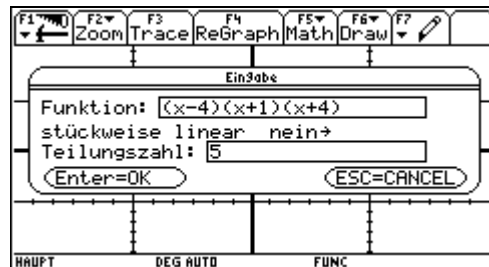
At first the program asks for the input of a function. We use a polynomial of third degree. At first we pay no attention to the other two points of the dialog box. The display is divided into four parts. On the left side in the upper part the graph of our function is shifted so that the value of the function on the left side of the displayed interval is zero. Also the tangent line on the left side of the displayed interval is drawn. On the right side in the lower part the graph of the derivative of our function is displayed.
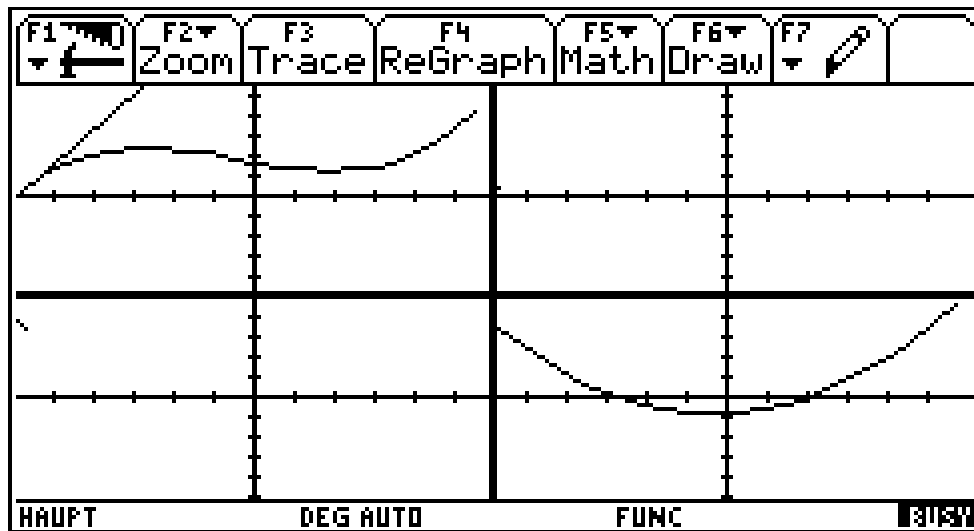
By using $\odot$ in the left part of the display the point where the tangent line is drawn moves to the right and in the lower part the graph of the derivative arises step by step. In the right part the area between the derivative and the x axis is shaded and in the upper part the graph of the integral function $\int\limits_{a}^{x} f$ arises step by step.
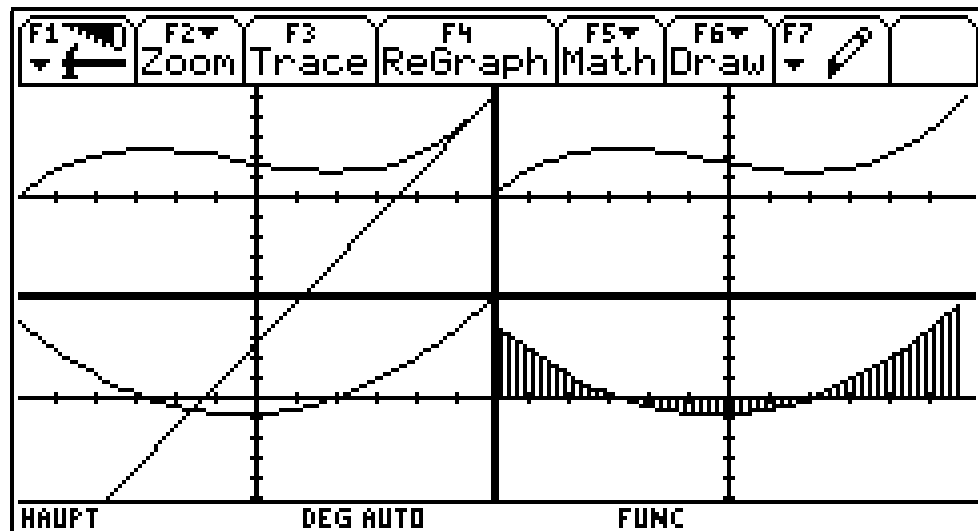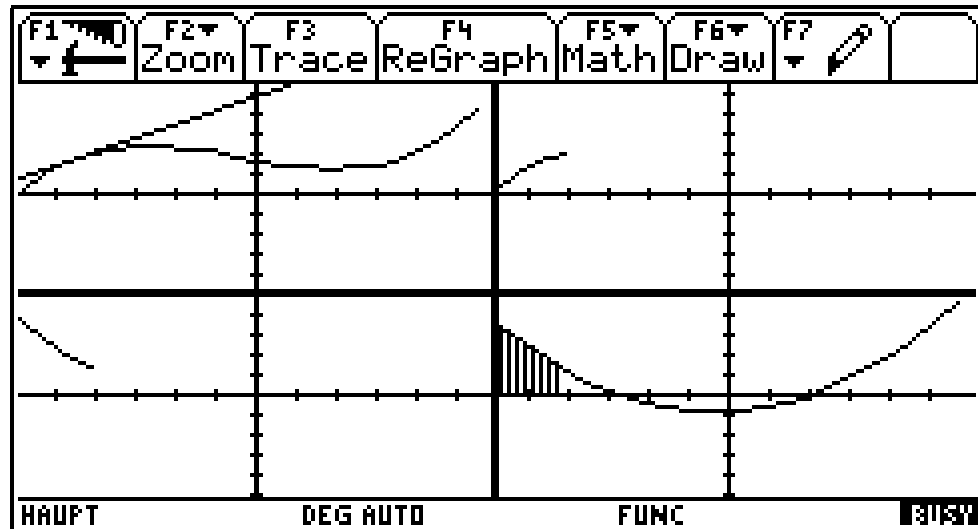
So pupils are able to see the inverse character of differentiation and integration.

Now the pupil should understand the connection between the calulation of the slope and the calculation of the area between the function and the x axis.

We start the program again and use a linear function. Again we pay no attention to the other two points of the dialog box.
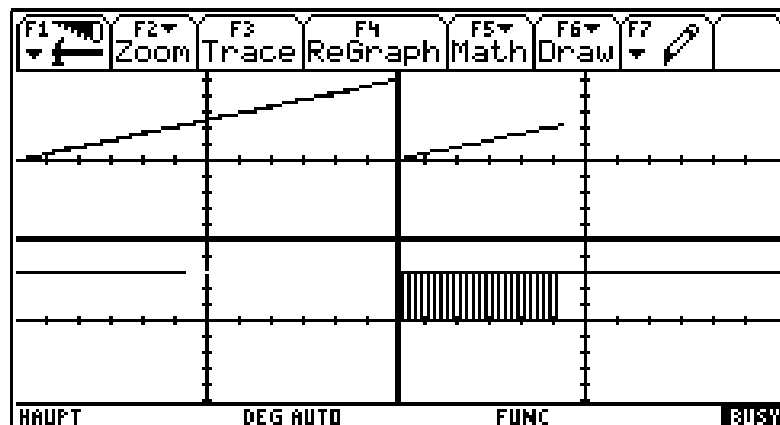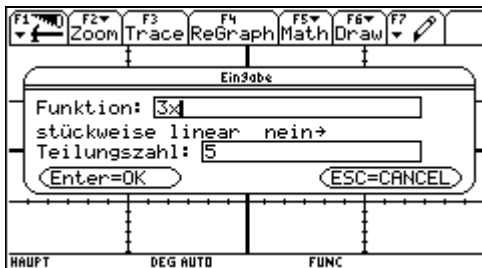
For a linear function the Fundamental Theory of Calculus is obvious. The derivative of a linear function is a constant function whose value is the slope of the linear function . The area between a constant function and the x axis grows linear with the slope equal to the constant value of the function. Now we start our program again and use the polynomial of third degree again.
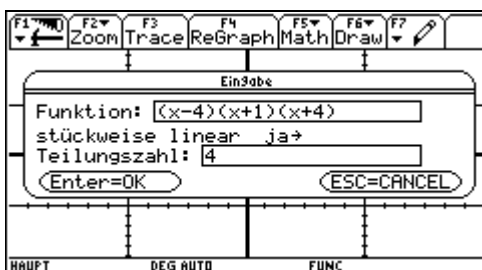
But now we use the other two points of the dialog box. We divide the displayed interval into four parts and replace the function by a piecewise linear function and the derivative by a piecewise constant function.
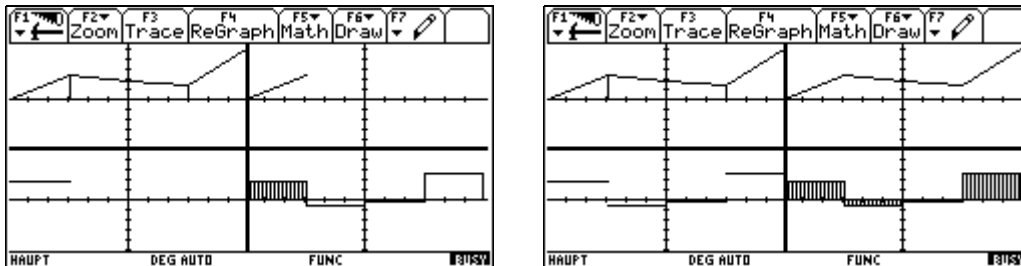
It is obvious, that Fundamental Theorem of Calculus is also correct for that function.
Now we start the program again and divide the interval into twenty parts.
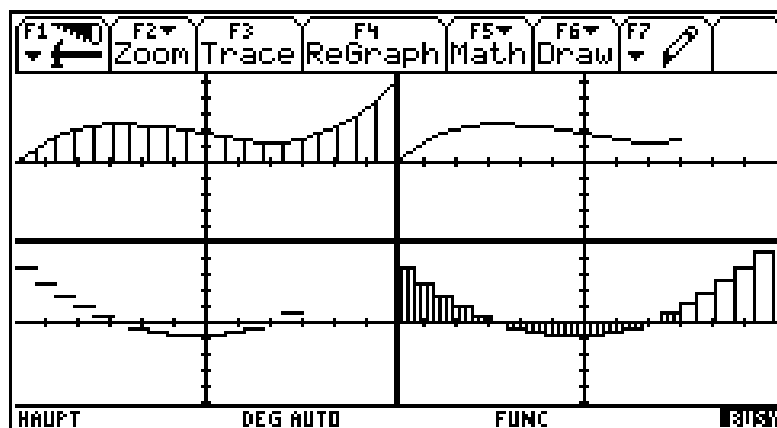
It is obvious that Fundamental Theorem of Calculus is also correct for that function.

Then we divide into forty parts.
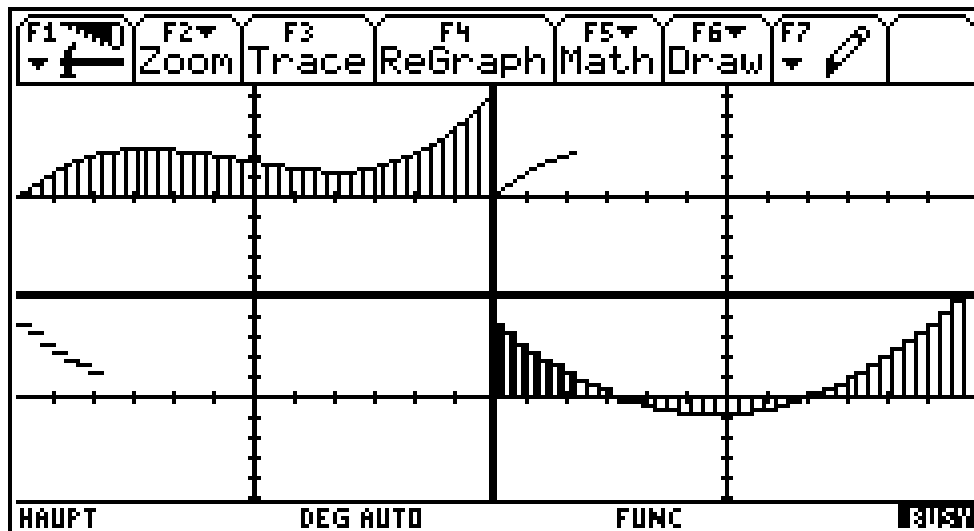
It is also obvious that Fundamental Theorem of Calculus is correct for that function. But the pupil sees that our polynomial of third degree can be replaced by a piecewise linear function as exactly one will.

This proof is not exact because it does not use ϵ and so on but it is obvious and that is important for pupils.

3.7) Conic sections

Conic sections are formed by intersection of a right circular double cone and a plane. Pupils often cannot visualize the intersection of a cone and a plane. This program shows a 3D-Graph of such an intersection. Pupils can modify position and form of the cone. The plane is the xy-plane. The axis of the cone is an element of the yz-plane. Also the equation of the conic section is calculated. After starting the program we must fix radius (Kegelradius) and the altitude (Kegelhöhe) of the cone at first. Then we must fix the angle between the axis of the cone and the xy-plane (Kegelneigung) and the z-intercept of the axis (Kegelposition).
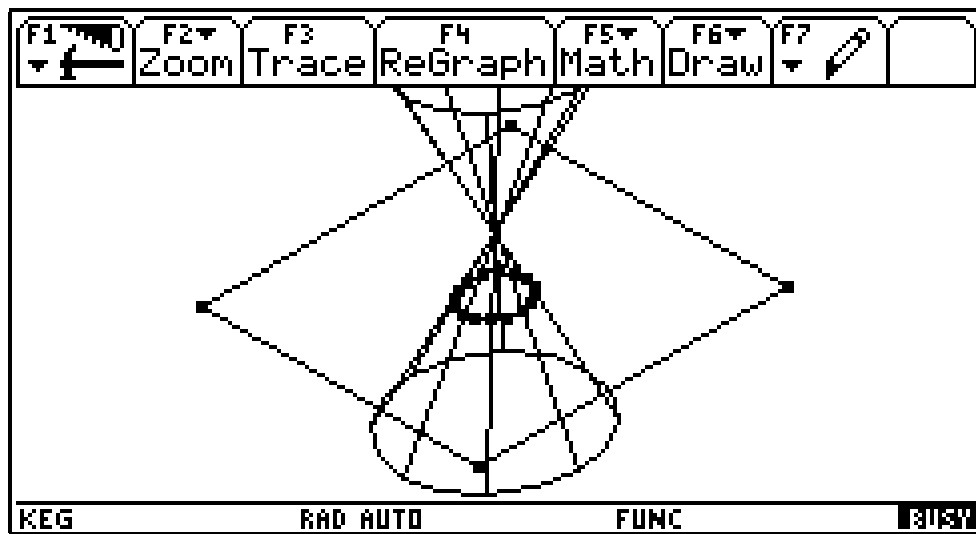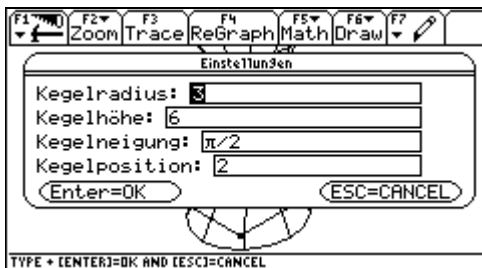
There are many possibilties of modifying the picture.
By pressing key l or shift l we can shift between a figure with or without
the cone.
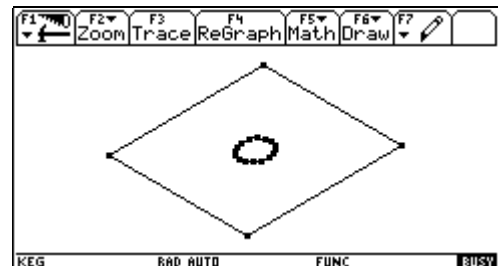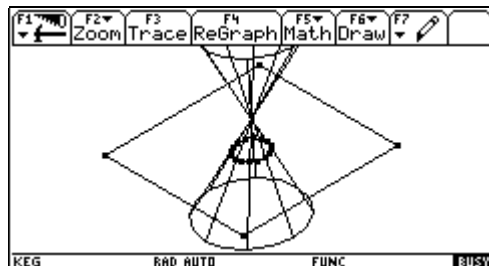
By pressing key g or key k we can make the viewing window larger or smaller.
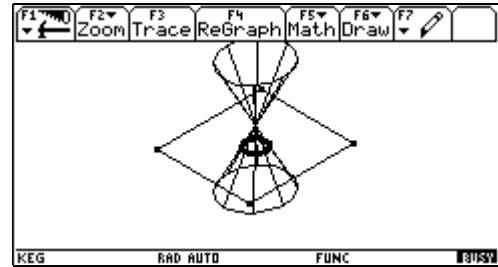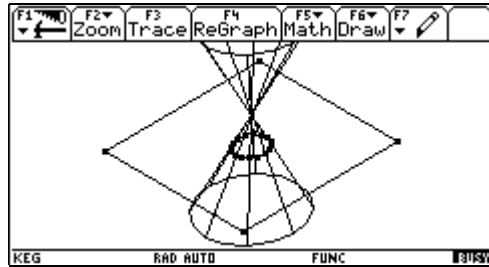
By pressing key w or shift w we can make the size of the representation of the xy-plane larger or smaller.
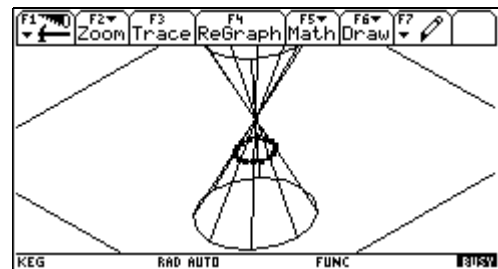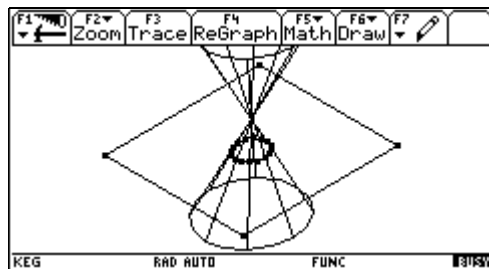
By pressing key x or y we can shift between the 3D-Graph and the Graph of the xy-plane with the conic section and its eqation.
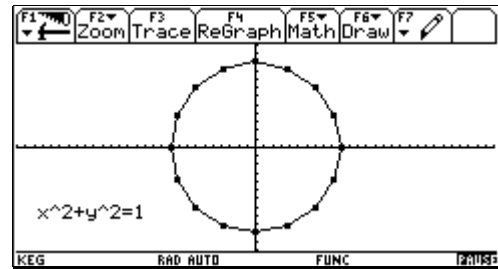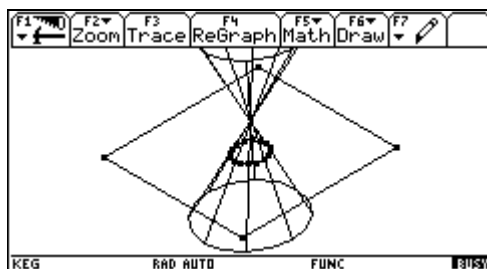
Now a short survey about all the other possiblities of intersection of a cone and a plane:

Hyperbola: The angle between the plane and the axis of the cone is smaller than the angle between an element of the cone and the axis, and the vertex of the cone is not a point of the plane.
For example the axis of the cone is parallel to the plane.
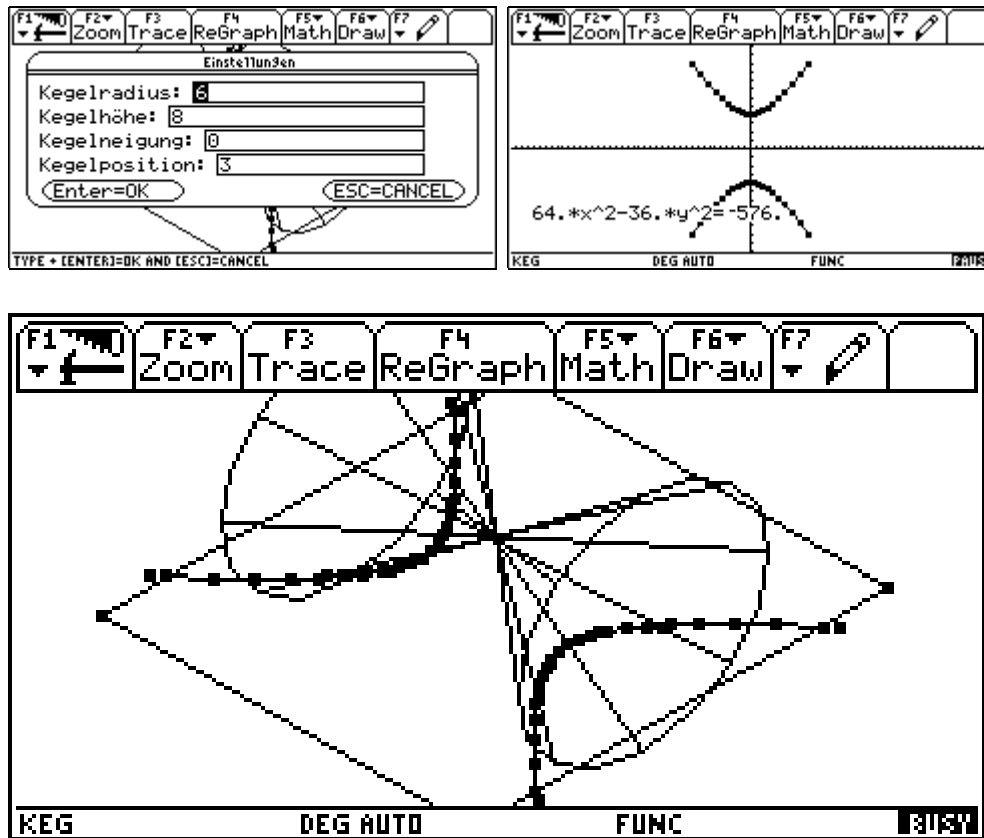
Two lines: The angle between the plane and the axis of the cone is smaller than the angle between an element of the cone, and the axis and the vertex of the cone is a point of the plane. For example the axis of the cone is an element of the plane
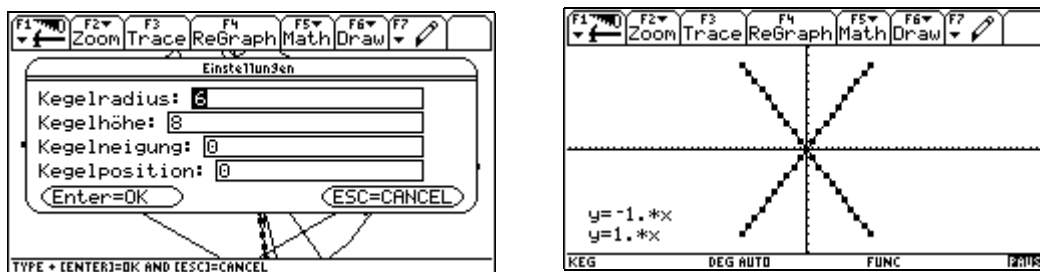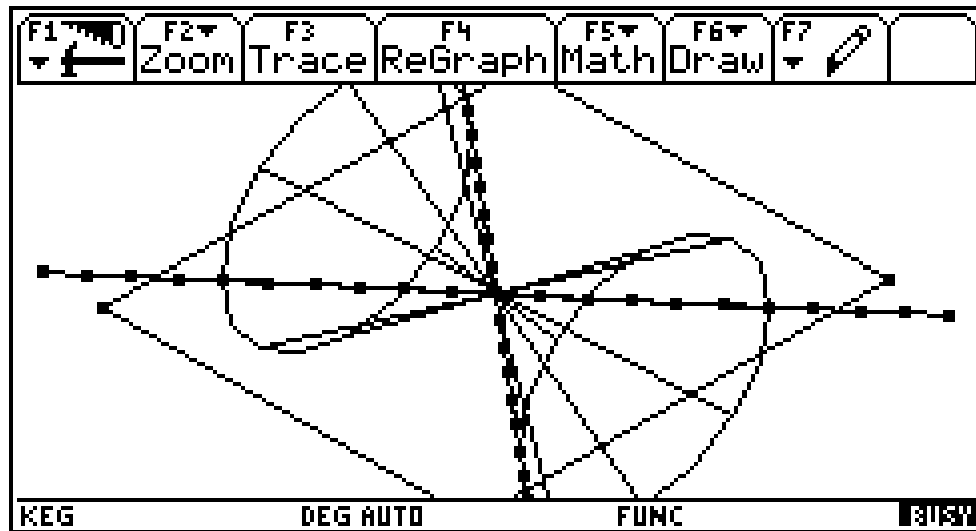
One line: The plane contacts the cone
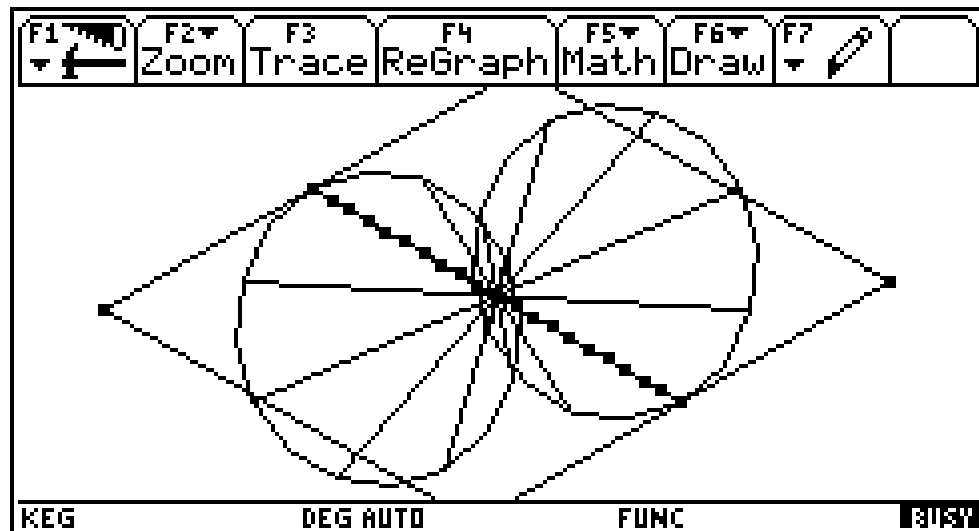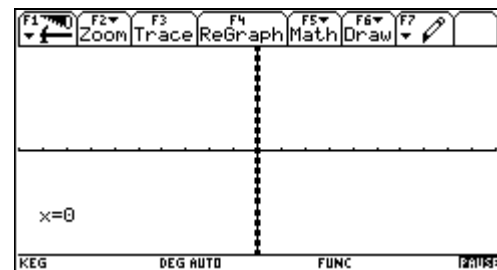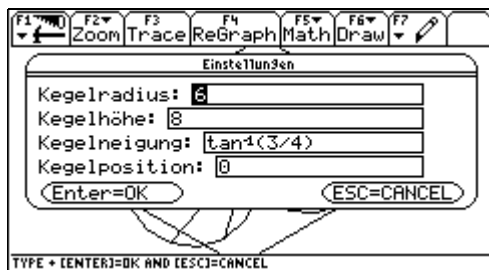
A parabola: The plane is parallel to an element of the cone

Kegelradius: 6
Kegelhöhe: 8
Kegelneigung: tan⁻¹(3/4)
Kegelposition: 2

$y = -3.\varepsilon{-1}*x^2 - 6.\varepsilon{-1}$
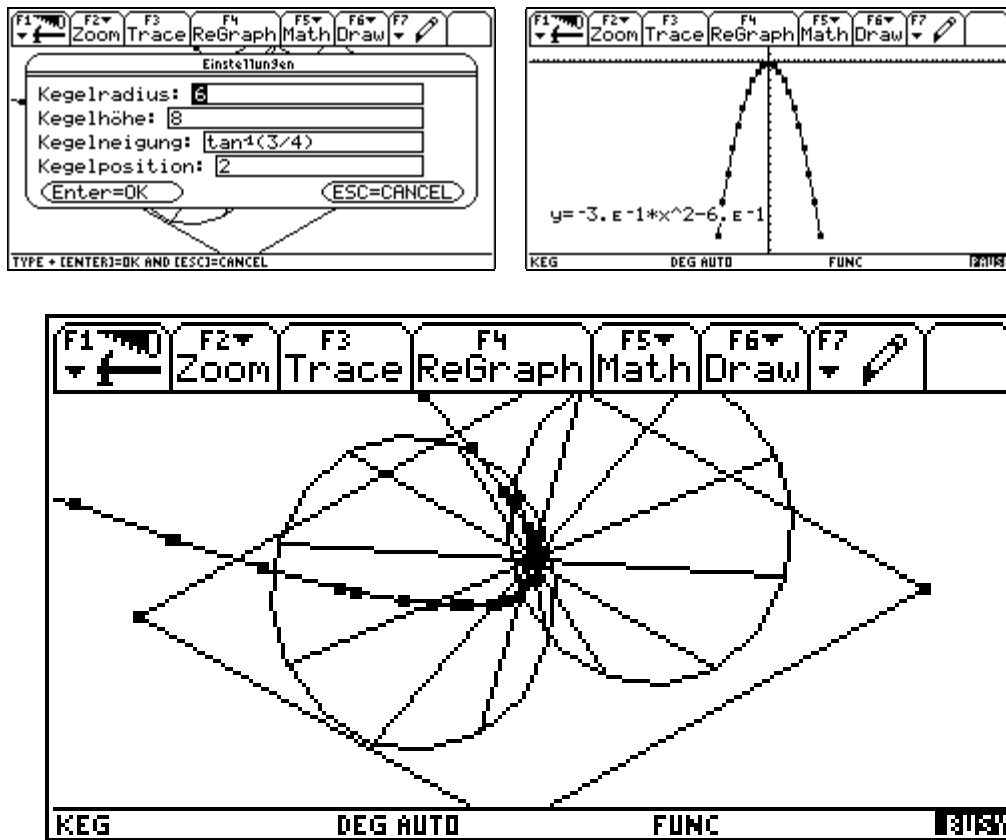
A circle: The axis of the cone is perpendicular to the plane and the vertex is not a point of the plane.

Kegelradius: 4
Kegelhöhe: 8
Kegelneigung: π/2
Kegelposition: 6

$x^2 + y^2 = 9$
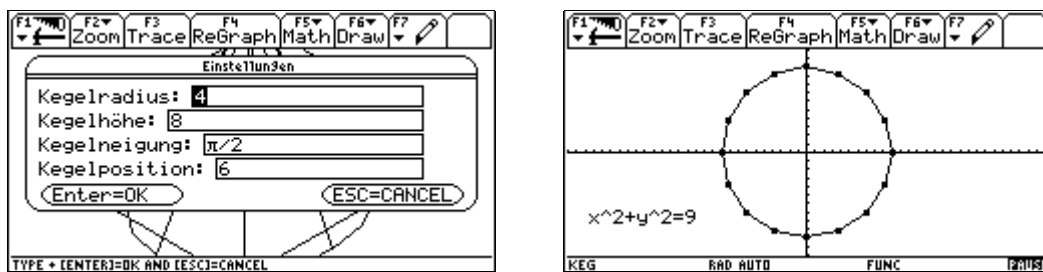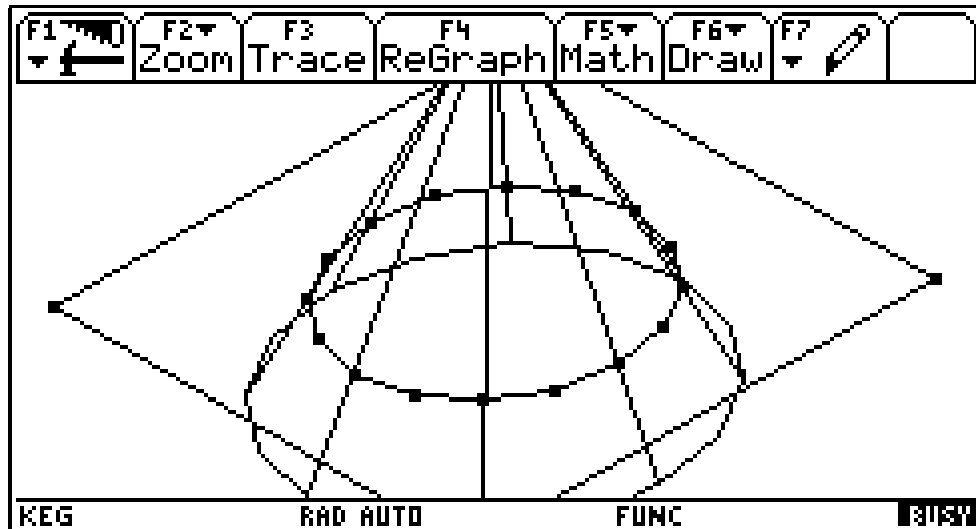
An ellipse: The angle between the plane and the axis of the cone ist bigger than the angle between an element of the cone and the axis, and the vertex is not a point of the plane.
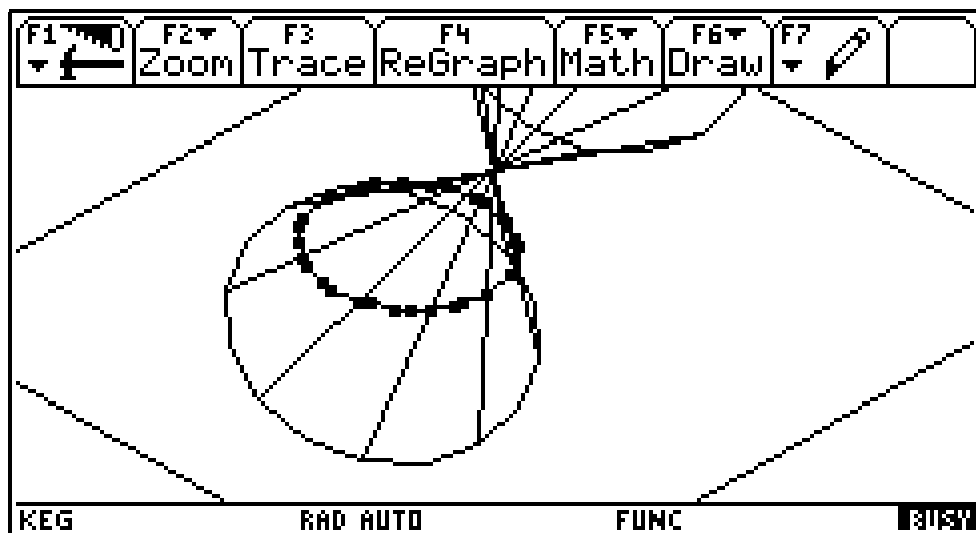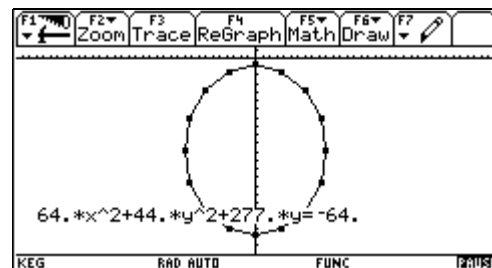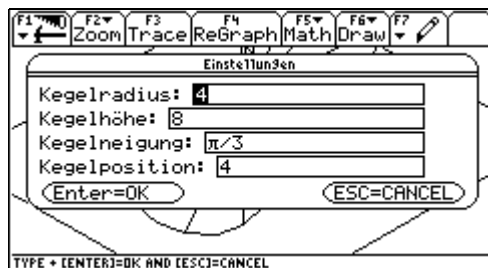
A point: The vertex of the cone is a point of the plane and the angle between the plane and the axis is bigger than the angle between an element of the cone and the axis.
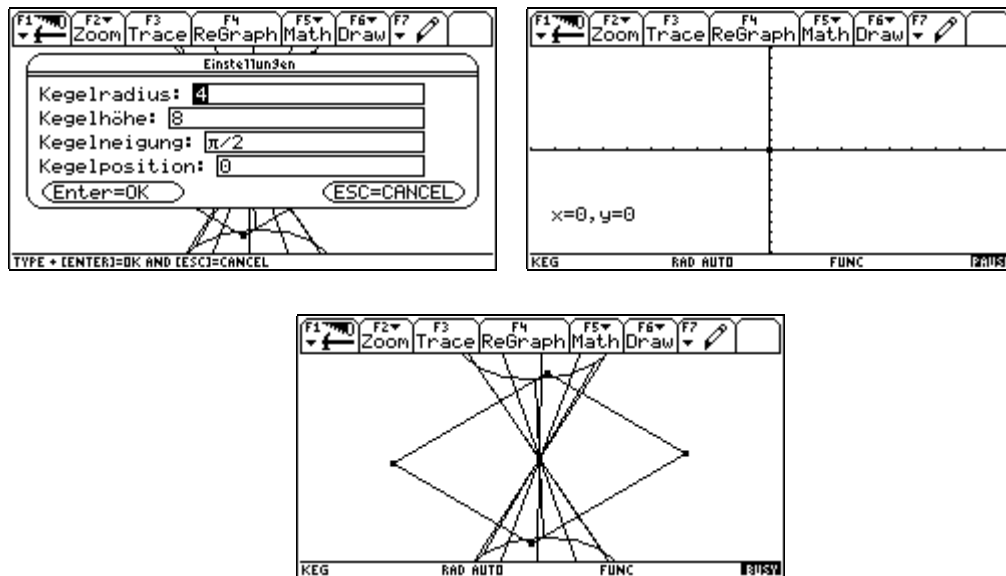
So the teacher can demonstrate the manifold possibilities of intersection by this program. Pupils can train their imaginative faculty. Also if there is time enough the pupils can try to find all possibilities by themselves. But that is a difficult exercise. The pupils can also study the differences between the equations of the conic sections.

3.8) Solid of revolution

Also this program should train the imaginative faculty of pupils. Often pupils cannot imagine the solid of revolution, which is generated by the revolution of the graph of a function around the x-axis.

After starting the program at first we must define our function and the lower bound (Untergrenze) and upper bound (Obergrenze) of the interval, for which the graph should be displayed. Then we can fix the number of the displayed circles (Anzahl der Kreise) and the number of the sides of the regular polygon (Anzahl der Ecken), which is displayed instead of the circle.

Then the graph will be displayed in the fixed interval. Lower and upper bound are displayed and the area between the graph and the x-axis is shaded.
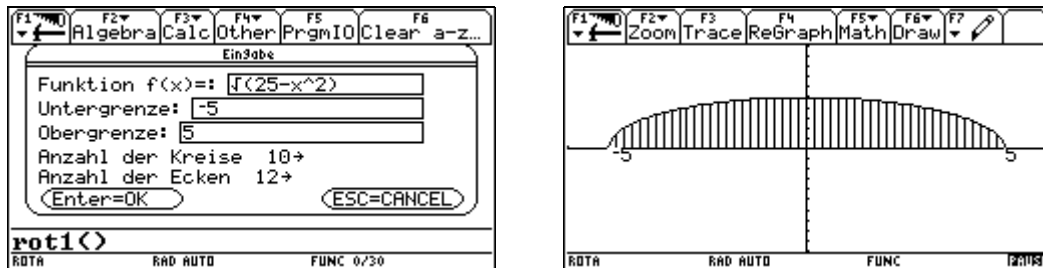
The next picture shows the x-Axis, the y-axis and the graph in a 3D-graphic.
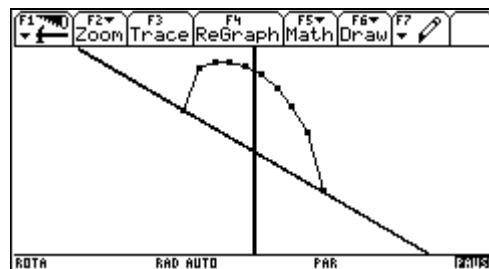
Then the program generates the solid of revolution. TI-92 must make many
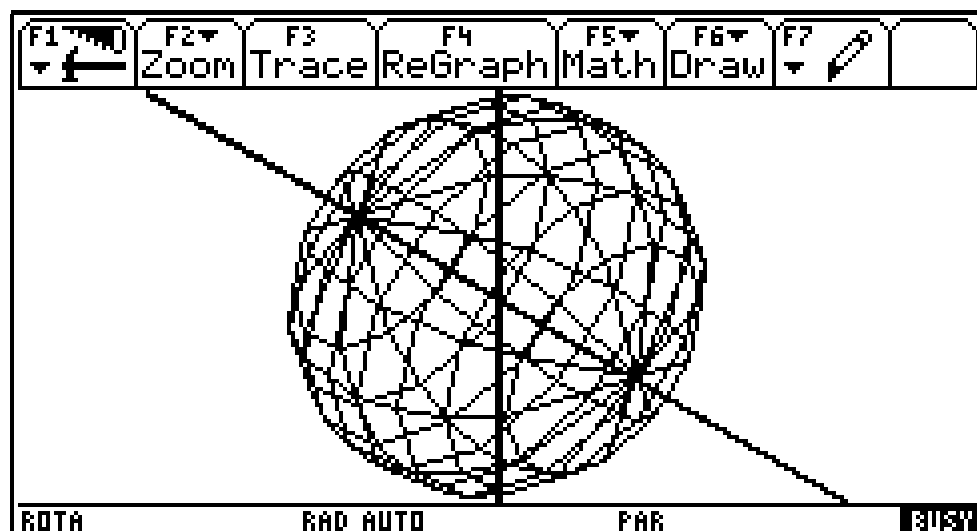
calculations for such a figure. So it takes one ore two minutes until the picture is finished.

By pressing key g or key k we can make the viewing window larger or smaller. This change does not take long, because all lines are made by plots and not by the drawing functions of the TI-92.
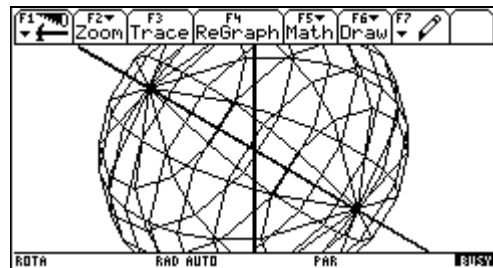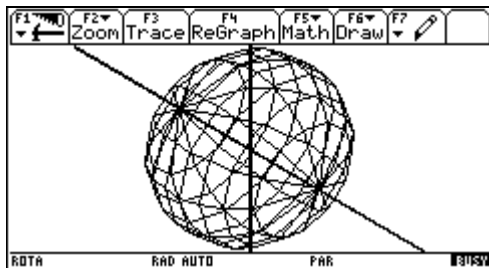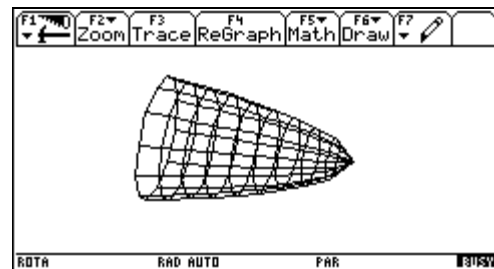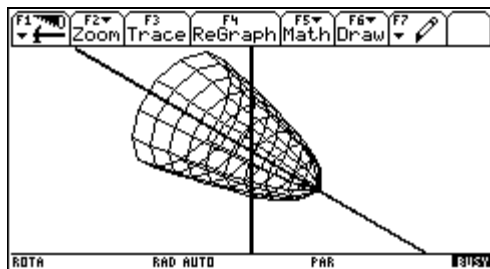

Then we will show some other examples.
First you see the solid of revolution of a linear function: a cone.
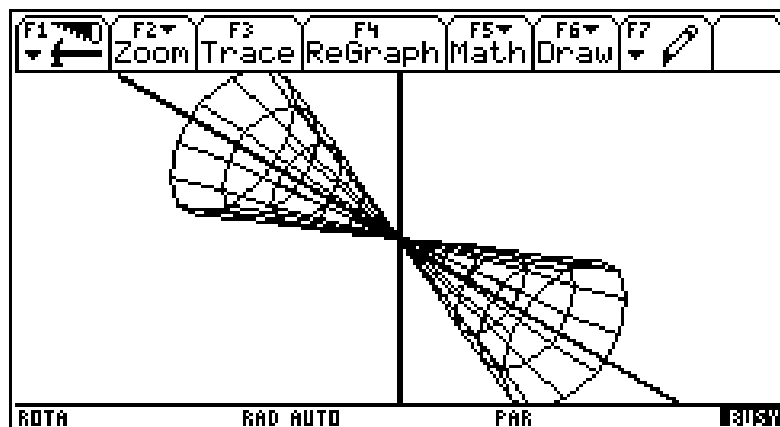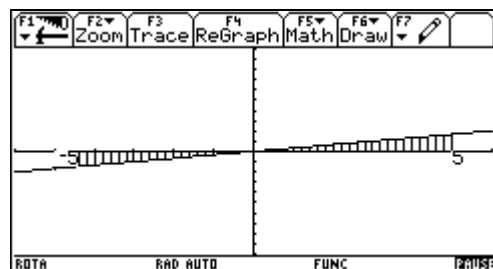
Then you see the solid of revolution of a root function: a parabolic surface.
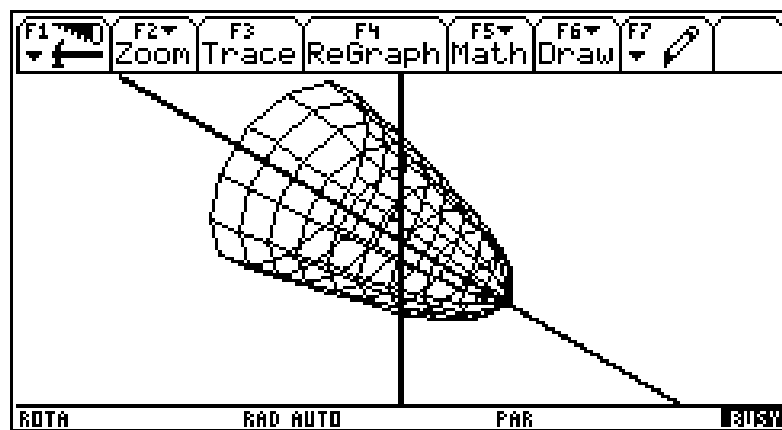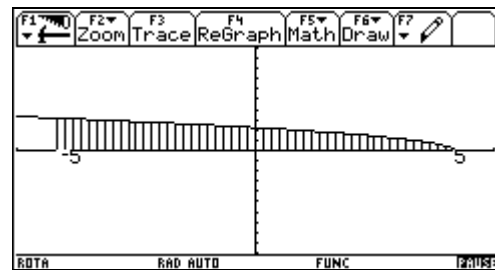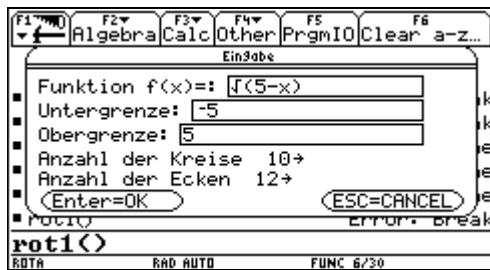
By pressing key d it is possible to rotate the solid of revolution around the y-axis.
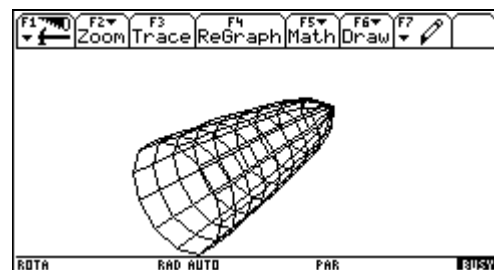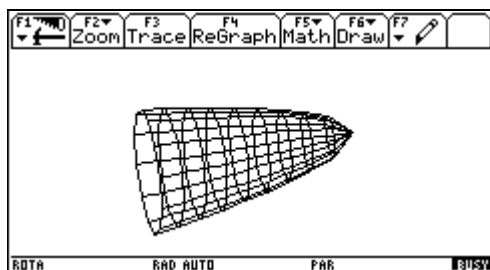
4) Self-reliant learning of subject matters

The aim of the following program is self-reliance in learning. When pupils have used this program long enough, they should be sure that they will recognize the term of the trigonometric function $f(x) = a \cdot \sin(\omega \cdot x + \Phi)$ by the graph of this function. They also should have learned to describe the influence of a or ω or ϕ on the graph of the function.
Now let us start the program. A new toolbar is displayed.

F1 Einführung: a playing interactive introduction to the properties of these functions.

F2 Lernprogramme: three programs, which teach you amplitude (Amplitude), angular frequency (Kreisfrequenz) and phaseshifting (Phasenverschiebung)
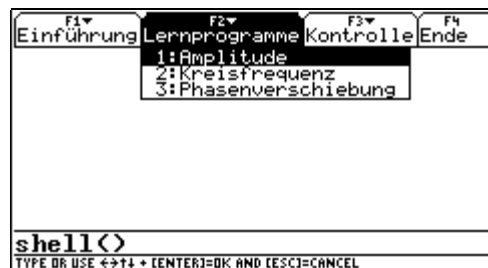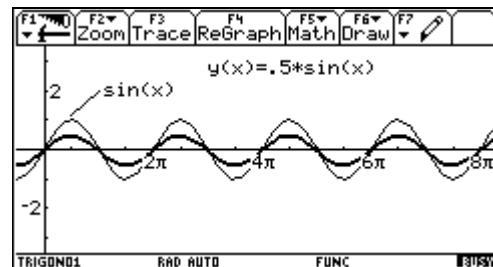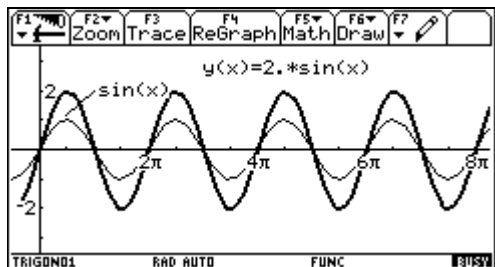
F3 Kontrolle: two programs for testing your knowledge in quality and in quantity

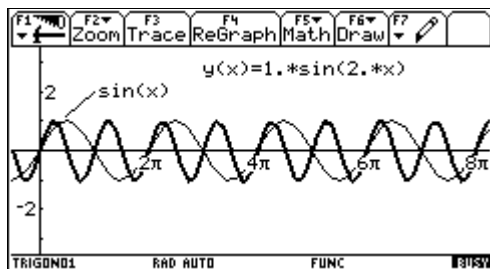
Now let us start the interactive introduction in a playful way.
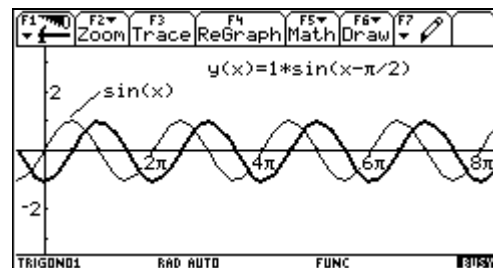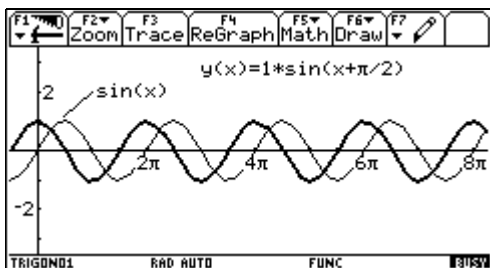The function f(x)=sin(x) is displayed.
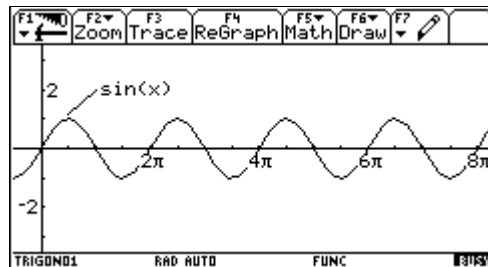
By pressing key a or shift a the amplitude will be changed in steps of 0.5 or - 0.5. The term of the modified function is displayed.

By pressing key w or shift w angular frequency will be changed in steps of 0.25 or - 0.25.
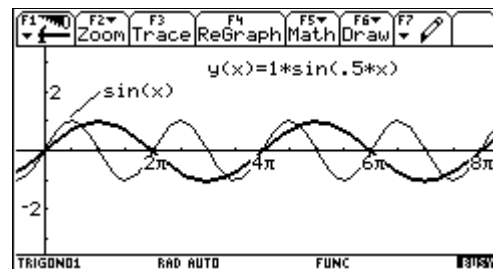
By pressing key p or shift p the phaseshifting will be changed in steps of π/2 or - π/2


We skip the programs for learning the importance of amplitude, angular frequency and phaseshifting, because the text is important in these programs but the text is in German. Therefore we start the program that tests our knowledge in qualitiy.

The graph of sinus and the graph of a function $f(x) = a \cdot \sin(\omega \cdot x + \Phi)$ are displayed. The function $f(x) = a \cdot \sin(\omega \cdot x + \Phi)$ is generated by random numbers.
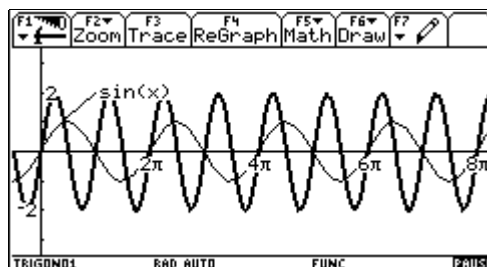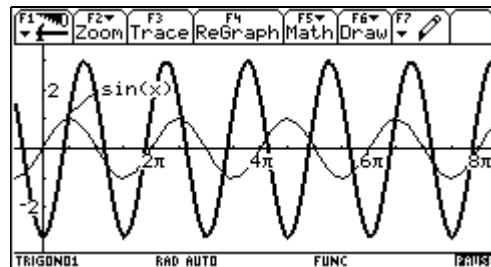

Now you will be asked if the amplitude of the thick graph is bigger or smaller than the amplitude of sinus, if angular frequency of the thick graph is bigger

or smaller than the angular frequency of sinus and if there is a phaseshifting between the thick graph and sinus. Then the program will control your answers. If an answer is wrong, you will have a chance again to see the graphs and correct your answer.
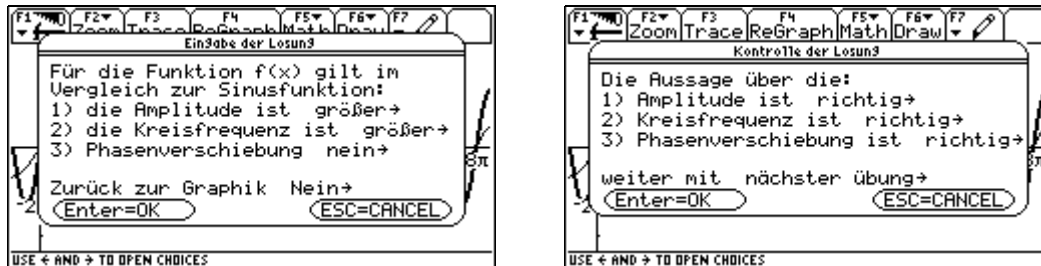

Now let us look at the program that tests our knowledge in quantity.
The graph of sinus and the graph of a function $f(x) = a \cdot \sin(\omega \cdot x + \Phi)$ are displayed. The function $f(x) = a \cdot \sin(\omega \cdot x + \Phi)$ is generated by random numbers.


You will be asked to determine the exact value of the amplitude, angular frequency and phaseshifting. The exactness of the picture of the graphs is not very big. Therefore in drop down menus the possible answers are displayed. Also this program will control your answers. If an answer is wrong, you will have a chance again to see the graphs and correct your answer.

Thomas Himmelbauer   34
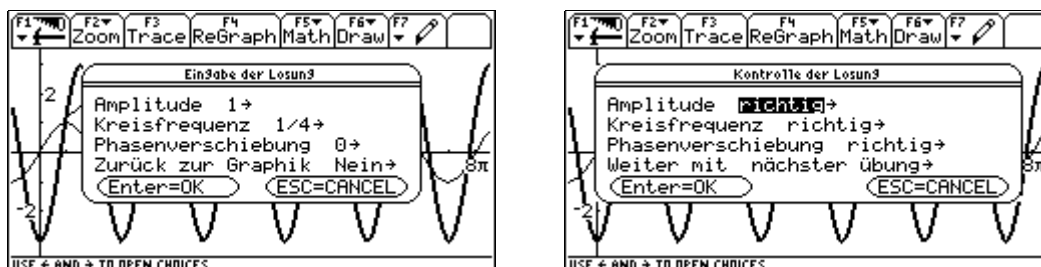
5.Exercise of well-known subject matters

5.1. Coordinates of a point

With this program pupils should exercise to determine the coordinates of a point in a coordinate system. The display of the TI-92 is bisected. On the left part the window variables are displayed, on the left part a coordinate system with a marked point ist shown. The coordinates of the point and the skale of x-axes and y-axes are generated by random numbers. The answers of the pupils are controlled by the program.
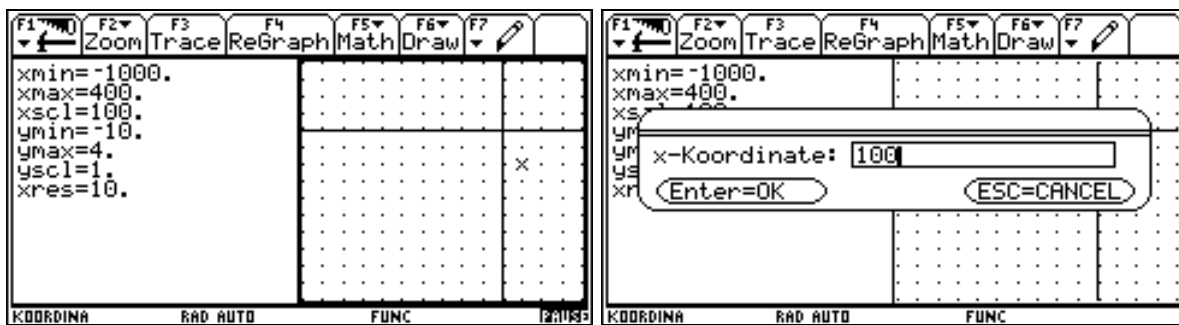
5.2. Slope and y-intercept of a line

With this program pupils should exercise to determine slope and y-intercept of a line. Slope and y-intercept of the line are generated by random numbers.
The answers of the pupils are controlled by the program.