Analytical Mechanics Problems with DERIVE

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I would like to present the work I have done in the last few years devoted to solving problems on General Physics (classical mechanics), with the help of computer algebra.

These problems are usually being solved during standard General Physics courses, at university level (engineering and science). Some of them would also be suitable for high schools.

Comparing various computer algebra systems (*Mathematica*, *Maple*, *Derive*, *MathCad*, *Reduce*) from the point of view of the comfort of their use, system requirements, price, applicability to physics specific problems etc. I have chosen the ***DERIVE***.

The result of this work (more than 130 solved problems) have been prepared for publication.

The majority of already published books, devoted to computer algebra systems use as illustrations purely mathematical, abstract problems. Only few of them explore physics problems, and to my knowledge no book similar to the one I have written has been published yet.

It is obvious that performing necessary symbolic derivations with the use of computer algebra system (CAS) reduces dramatically the time of work and allows a student (or teacher) to concentrate on physical ideas (which is most important), rather then on very time consuming technical side - performing the derivations by hand.

Each problem of the book is splited into few section:

(a) the problem content,
(b) description of the common methods of its solution (by hand) including all formulae necessary for the solution of the problem,
(c) the form in which the relations can be entered to the computer
(d) the view of a screen that after all previous steps and commands have been done. (In most of problems the picture have been splited into few pictures to present results more clearly).

In many problems more than one method leading to its solution is presented. In such cases the last method is the one which uses the power of ***DERIVE*** on its highest level.

The limited time allows me to present only few examples from this book. Therefore I will concentrate on examples in which some shortcomings of *DERIVE* appear.

Let us start with very simple example

Example 1
The trajectory of a moving point is given in the parametrised form:

$$x = Vo\cos(\alpha)t, \quad y = Vo\sin(\alpha)t - \frac{gt^2}{2} .$$

Evaluate tangential and normal components of acceleration of this point and the curvature radius of the trajectory.

Solution: We enter:
 (a) position vector
 (b) velocity vector:
 (c) modulae of the velocity:
 (d) tangential and normal components of the acceleration and curvature radius:

$$\#1:\quad r\_ := \left[x := Vo\cdot t\cdot COS(\alpha),\ y := Vo\cdot SIN(\alpha)\cdot t - \frac{g\cdot t^2}{2}\right]$$

$$\#2:\quad V\_ := \frac{d}{dt}\, r\_$$

$$\#3:\quad V := |V\_|$$

$$\#4:\quad as := \frac{d}{dt}\, V$$

$$\#5:\quad an := \sqrt{(g^2 - as^2)}$$

$$\#6:\quad rho := \frac{V^2}{an}$$

In the expression defining **an** we have used the fact that the total acceleration is equal to **g** (this can be easily verified).

Simplification of **#4**, **#5** and **#6** (strickly said **RHS** of **#4, #5 and #6**) leads to the results:

$$\#7:\quad as := \frac{g\cdot(g\cdot t - Vo\cdot SIN(\alpha))}{\sqrt{(-2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) + Vo^2 + g^2\cdot t^2)}}$$

$$\#8:\quad an := -\frac{|Vo\cdot g\cdot COS(\alpha)|\cdot SIGN(2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) - Vo^2 - g^2\cdot t^2)}{\sqrt{(-2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) + Vo^2 + g^2\cdot t^2)}}$$

$$\#9:\quad rho := -\frac{SIGN(2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) - Vo^2 - g^2\cdot t^2)\cdot(-2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) + Vo^2 + g^2\cdot t^2)^{3/2}}{|Vo\cdot g\cdot COS(\alpha)|}$$

To make **#8** and **#9** simpler (to get rid of modulus) we declare the domains for **Vo**, **g** and α. For **an** we get

$$\#12:\quad [Vo :\in Real\ (0, \infty), g :\in Real\ (0, \infty), \alpha :\in Real\ (0, \pi/2)]$$

$$\#13:\quad an := -\frac{Vo\cdot g\cdot |COS(\alpha)|\cdot SIGN(2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) - Vo^2 - g^2\cdot t^2)}{\sqrt{(-2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) + Vo^2 + g^2\cdot t^2)}}$$

Unfortunately the function **SIGN** still remains (**#13**). However, it can be easily checked that the argument of **SIGN** is always negative because it is the second order polynomial with no real zeros and branches going downward. We prove it by solving the equation **#14**:

$$\#14:\quad 2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) - Vo^2 - g^2\cdot t^2 = 0$$

$$\#15:\quad t = \frac{Vo\cdot SIN(\alpha)}{g} + \frac{\hat{\imath}\cdot Vo\cdot |COS(\alpha)|}{g}$$

$$\#16:\quad t = \frac{Vo\cdot SIN(\alpha)}{g} - \frac{\hat{\imath}\cdot Vo\cdot |COS(\alpha)|}{g}$$

Solutions **#15, #16** are complex. We can replace then the function **SIGN** with '**-1**'. For **an** we get:

$$\#20:\quad an := -\frac{Vo\cdot g\cdot |COS(\alpha)|\cdot(-1)}{\sqrt{(-2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) + Vo^2 + g^2\cdot t^2)}}$$

$$\#21:\quad an := \frac{Vo\cdot g\cdot |COS(\alpha)|}{\sqrt{(-2\cdot Vo\cdot g\cdot t\cdot SIN(\alpha) + Vo^2 + g^2\cdot t^2)}}$$

Note:

- Let us notice that that ***DERIVE*** 3.11 can not simplify the function $|\cos(\alpha)|$ for the declared domain $0<\alpha<\pi/2$. This remark is **not true** for later versions of the program.

Example 2

Prove the general statement that any central force field is potential.

Solution: The field is potential if *curl* of the force field is equal to zero

$$curl\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ F_x & F_y & F_z\end{vmatrix}=0,\qquad F_x,\ F_y,\ F_z$$ - components of the force.

Central force can be written in the form $\vec{F}=f(r)\vec{r}$,
where:

$\vec{r}$ - position vector,

$f(r)$ - arbitrary differentiable function of the magnitude of the position vector.

To prove that the central field is potential we enter:

```
#1:   r_ := [x, y, z]

#2:   r := |r_|

#3:   f(r) :=

#4:   F:=f(r)·r_
```

and evaluate one by one the components of the *curl* vector. For the *x* component we get:

```
#5:  ELEMENT(CURL(F), 1)
```

$$\#6:\quad \frac{y\cdot z\cdot \lim\limits_{@7\to\sqrt{(x^2+y^2+z^2)}} \frac{d}{d\,@7} f(@7)}{\sqrt{(x^2+y^2+z^2)}} - \frac{y\cdot z\cdot \lim\limits_{@8\to\sqrt{(x^2+y^2+z^2)}} \frac{d}{d\,@8}}{\sqrt{(x^2+y^2+z^2)}}$$

We can see, that the result **#6** (simplification of **#5**), is larger than the width of the window. In this case however it is enough to use compressed format mode (see **#7**) to overcome this inconvenience. Under this mode the ***DERIVE*** displays the result **#6** in the form **#8**:

#7: DisplayFormat:=Compressed

#8: $$\frac{y\cdot z\cdot \lim\limits_{@7\to\sqrt{(x^2+y^2+z^2)}} \frac{d}{d\,@7} f(@7)}{\sqrt{(x^2+y^2+z^2)}} - \frac{y\cdot z\cdot \lim\limits_{@8\to\sqrt{(x^2+y^2+z^2)}} \frac{d}{d\,@8} f(@8)}{\sqrt{(x^2+y^2+z^2)}}$$

To show that **#8** is zero we have to replace names of arbitrary variables **@7**, **@8** with one name, let us say **@8**. In this way we get **#9**, which simplified results in **#10**, as expected:

#9: $$\frac{y\cdot z\cdot \lim\limits_{@8\to\sqrt{(x^2+y^2+z^2)}} \frac{d}{d\,@8} f(@8)}{\sqrt{(x^2+y^2+z^2)}} - \frac{y\cdot z\cdot \lim\limits_{@8\to\sqrt{(x^2+y^2+z^2)}} \frac{d}{d\,@8} f(@8)}{\sqrt{(x^2+y^2+z^2)}}$$

#10: 0

To complete the prove of the potential character of the examined field, we
need to do similar calculations for the remaining components of the field.

Remarks:

- Unfortunately the assignment of the form a:= it is not accepted.
- The later version of **DERIVE (v. 4.09)** solves the problem. immediately:

```
#1:   r_:=[x,y,z]
#2:   r:=|r_|
#3:   f(r):=
#4:   F:=f(r)·r_
#5:   CURL(F)
#6:   [0,0,0]
```

The vector **#6** is the simplification result of **#5**.

The solution of the problem can be obtained immediately if we employ spherical coordinates

$$curl\vec{F} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} e_r & e_\theta r & e_\varphi r \sin\theta \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ F_r & rF_\theta & rF_\varphi \sin\theta \end{vmatrix}$$

where: $F_\theta = F_\varphi = 0,\ F_r = f(r)\ /|\vec{r}| = g(r)$

Example 3
A ring is made of a thin smooth pipe (thin torus). There is a piece of wire inside the pipe which can move without friction (Fig. 1). The ring is rotating with an angular velocity ω around an axis of symmetry lying in the plane of the ring. Evaluate and discuss the equilibrium positions of the wire for two cases, when the length of the wire is equal:

a) half of the ring,
b) forth of the ring.

Solution: Let us consider the wire element of the length *dl*. It is convenient to treat the problem in the non-inertial reference system connected with the rotating ring. In such system the wire undergoes the action of two forces, gravitational and centrifugal one. The force fields are conservative and can be described by respective potentials.

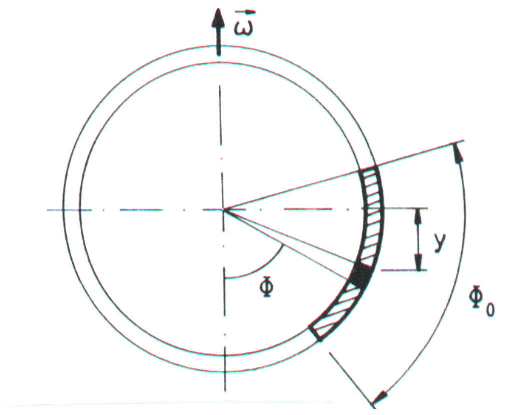

Fig.1.

The energy of the mass element is the sum of the gravitational potential energy

$$dE_g = -dmgy = -\sigma g R^2 \cos\phi d\phi, \quad (dm = \sigma R d\phi)$$

and the potential energy of the centrifugal force:

$$dE_c = -\frac{1}{2}\omega^2 r^2 dm = -\frac{1}{2}\omega^2 R^3 \sin^2\phi d\phi$$

The total energy is just the integral over the whole wire which, in our notation, is represented by the integral over the whole angle Φ_0

$$E_p = \int_{\Phi}^{\Phi+\Phi_0} dE_g + \int_{\Phi}^{\Phi+\Phi_0} dE_c$$

At the equilibrium the energy E_p is extremal. This means that the lower limit of integration (ϕ) should satisfy the condition:

$$\frac{dE_p}{d\Phi} = 0$$

The type of equilibrium follows from the sign of the second derivative:

$$\frac{d^2 E_p}{d\Phi^2}$$

We enter then the expressions:

$$\#1:\quad Ep := \int_{\Phi}^{\Phi + \Phi o} \left[-\sigma \cdot g \cdot R^{2} \cdot COS(\Phi) - \frac{1}{2} \cdot \sigma \cdot w^{2} \cdot R^{3} \cdot SIN(\Phi)^{2} \right] d\Phi$$

$$\#2:\quad \frac{d}{d\Phi} Ep = 0$$

a) In this case the length of the wire equals half of the tube length, $\phi o=\pi$. Simplification of **#2** returns equation **#4**. Its solutions are displayed in **#5**, **#6** and **# 7**

$$\#3:\quad \Phi o := \pi$$

$$\#4:\quad 2 \cdot R^{2} \cdot g \cdot \sigma \cdot COS(\Phi) = 0$$

$$\#5:\quad \Phi = \frac{\pi}{2}$$

$$\#6:\quad \Phi = -\frac{\pi}{2}$$

$$\#7:\quad \Phi = \frac{3 \cdot \pi}{2}$$

The angles **#5** and **#6** describe the equivalent configurations, therefore for further calculations we will use only one of them and the **#7**. Entering the expressions:

ϕ := π/2
SIGN(DIF(Ep, PHI, 2) =

we get **#8** and **#9**, and after the declarations **#10**, we get **#11**:

$$\#8:\quad \Phi := \frac{\pi}{2}$$

$$\#9:\quad SIGN\left[\left[\frac{d}{d\Phi}\right]^{2} Ep\right] = -SIGN(g \cdot \sigma)$$

$$\#10:\quad [g :\in Real\ (0, \infty),\ \sigma :\in Real\ (0, \infty)]$$

$$\#11:\quad SIGN\left[\left[\frac{d}{d\Phi}\right]^{2} Ep\right] = -1$$

which means that at the angle $\pi/2$ the equilibrium is unstable.

For the second angle we obtain:

$$\#12:\quad \Phi := \frac{3}{2}\cdot\pi$$

$$\#13:\quad \mathrm{SIGN}\left[\left[\frac{d}{d\Phi}\right]^2 Ep\right] = 1$$

so, the equilibrium at the angle $3\pi/2$ is stable.

b) The simplification of the equation **#2** but for **$\Phi o=\pi/2$** returns the equation **#15**

$$\#14:\quad \left[\Phi :=,\ \Phi o := \frac{\pi}{2}\right]$$

$$\#15:\quad -R^3\cdot\sigma\cdot w^2\cdot\mathrm{COS}(\Phi)^2 + R^2\cdot g\cdot\sigma\cdot\mathrm{COS}(\Phi) + R^2\cdot g\cdot\sigma\cdot\mathrm{SIN}(\Phi) + \frac{R^3\cdot\sigma\cdot w^2}{2} = 0$$

which is too difficult for *DERIVE*. We have to perform some preliminary calculations by hand. The equation **#15** divided by $R^2\sigma$ gives **#17**:

$$\#16:\quad \frac{-R^3\cdot\sigma\cdot w^2\cdot\mathrm{COS}(\Phi)^2 + R^2\cdot g\cdot\sigma\cdot\mathrm{COS}(\Phi) + R^2\cdot g\cdot\sigma\cdot\mathrm{SIN}(\Phi) + \frac{R^3\cdot\sigma\cdot w^2}{2} = 0}{R^2\cdot\sigma}$$

$$\#17:\quad -R\cdot w^2\cdot\mathrm{COS}(\Phi)^2 + g\cdot\mathrm{COS}(\Phi) + g\cdot\mathrm{SIN}(\Phi) + \frac{R\cdot w^2}{2} = 0$$

It is still untreatable for *DERIVE*. However it can be rewritten (by hand) in the form **#18**

$$\#18:\quad \mathrm{SIN}(\Phi) + \mathrm{COS}(\Phi) = C\cdot\mathrm{COS}(2\cdot\Phi)$$

#19: "C=w^2*R/2g"

and next in the form **#21**:

$$\#21:\quad \mathrm{SIN}(\Phi) + \mathrm{COS}(\Phi) = C\cdot(\mathrm{COS}(\Phi) - \mathrm{SIN}(\Phi))\cdot(\mathrm{SIN}(\Phi) + \mathrm{COS}(\Phi))$$

The equation splits into two equations:

$$\sin\Phi + \cos\Phi = 0 \text{ or } C(\cos\Phi - \sin\Phi) = 1,$$

where $C = \dfrac{\omega^2 R}{2g}$.

For the first of these two equations we get solutions **#23, #24** and **#25**:

$$\#22:\ \mathrm{SIN}(\Phi) + \mathrm{COS}(\Phi) = [illegible]$$

$$\#23:\ \Phi = -\frac{\pi}{4}$$

$$\#24:\ \Phi = \frac{3\cdot\pi}{4}$$

$$\#25:\ \Phi = -\frac{5\cdot\pi}{4}$$

Solutions **#24** and **#25** denote the same wire position.

For the second equation ***DERIVE*** returns solutions **#27**, **#28** and **#29**

$$\#26:\ [illegible] = C\cdot(\mathrm{COS}(\Phi) - \mathrm{SIN}(\Phi))$$

$$\#27:\ \Phi = \mathrm{ASIN}\left[\frac{\sqrt{2}}{2\cdot C}\right] - \frac{3\cdot\pi}{4}$$

$$\#28:\ \Phi = \mathrm{ASIN}\left[\frac{\sqrt{2}}{2\cdot C}\right] + \frac{5\cdot\pi}{4}$$

$$\#29:\ \Phi = \frac{\pi}{4} - \mathrm{ASIN}\left[\frac{\sqrt{2}}{2\cdot C}\right]$$

The angles **#27** and **#28** denote the same position of the wire. Therefore the analysis of the equilibrium type can be restricted to the angles **#23, #24, #27** and **#29**

The arguments of **ASIN** must satisfy the obvious inequality:

$$-1 \le \frac{\sqrt{2}}{2C} \le 1, \quad \text{where} \quad C = \frac{\omega^2 R}{2g}.$$

which leads to the inequality for the angular velocity ω.

Let us find the ω:

$$\#30:\quad C := \frac{w^{2}\cdot R}{2\cdot g}$$

$$\#31:\quad R :\in \text{Real } (0, \infty)$$

$$\#32:\quad \frac{\sqrt{2}}{2\cdot C} < 1$$

$$\#33:\quad w > \frac{2^{1/4}\cdot\sqrt{g}}{\sqrt{R}}$$

$$\#34:\quad w < -\frac{2^{1/4}\cdot\sqrt{g}}{\sqrt{R}}$$

We have found the frequencies for which equilibrium states can exist at the angles **#27, #28** and **#29**.

We enter the obtained angles and check the signs of the second derivatives using matrix notation:

$$\#35:\quad \left[\Phi1 := -\frac{\pi}{4},\ \Phi2 := \frac{3}{4}\cdot\pi,\ \Phi3 := \text{ASIN}\left[\frac{\sqrt{2}}{2\cdot C}\right] - \frac{3\cdot\pi}{4},\ \Phi4 := \frac{\pi}{4} - \text{ASIN}\left[\frac{\sqrt{2}}{2\cdot C}\right]\right]$$

$$\#36:\quad \begin{bmatrix} \Phi := \Phi1 & \text{SIGN}\left[\left[\frac{d}{d\Phi}\right]^{2} Ep\right] \\ \Phi := \Phi2 & \text{SIGN}\left[\left[\frac{d}{d\Phi}\right]^{2} Ep\right] \\ \Phi := \Phi3 & \text{SIGN}\left[\left[\frac{d}{d\Phi}\right]^{2} Ep\right] \\ \Phi := \Phi4 & \text{SIGN}\left[\left[\frac{d}{d\Phi}\right]^{2} Ep\right] \end{bmatrix} = \begin{bmatrix} -\frac{\pi}{4} & -\text{SIGN}(R\cdot w^{2} - \sqrt{2}\cdot g) \\ \frac{3\cdot\pi}{4} & -1 \\ \text{ASIN}\left[\frac{\sqrt{2}\cdot g}{R\cdot w^{2}}\right] - \frac{3\cdot\pi}{4} & \text{SIGN}(R^{2}\cdot w^{4} - 2\cdot g^{2}) \\ \frac{\pi}{4} - \text{ASIN}\left[\frac{\sqrt{2}\cdot g}{R\cdot w^{2}}\right] & \text{SIGN}(R^{2}\cdot w^{4} - 2\cdot g^{2}) \end{bmatrix}$$

We find the results of simplification in the rows of the matrix on the right side of the equality sign in **#36**. In the first column there are the angles and in the second column there are the **SIGN** functions.

Let us discuss the results.

- At the angle **Φ2,** the equilibrium is unstable.
- At the angles **Φ3** and **Φ4** the equilibrium exists if the condition **#33** is fulfilled and is stable.
- At the angle **Φ1** the equilibrium is unstable if the condition **#33** is fulfilled, otherwise stable.

Example 4

A particle of mass *m* is moving in the force field of the potential energy

$$E_p = -\alpha \frac{m}{r^n}, \text{ where } \alpha = const > 0.$$

Evaluate the range of the parameter *n* for which there exist stable orbit.

Solution: Energy of the particle is the sum of its kinetic and potential energy

$$E = \frac{L^2}{2mr^2} - \alpha \frac{m}{r^n},$$

where L is angular momentum of the particle.
The necessary condition for the existence of the stable orbit is that the total energy, as a function of r, of the particle has the minimum.
From the mathematical point of view the condition can be formulated as

$$\frac{\partial E}{\partial r} = 0 \text{ and } \frac{\partial^2 E}{\partial r^2} > 0.$$

We enter the data:

$$\#1: \quad E := \frac{L^2}{2 \cdot m \cdot r^2} - \frac{\alpha \cdot m}{r^n}$$

$$\#2: \quad \alpha :\in \text{Real } (0, \infty)$$

and the equation **#3**

$$\#3: \quad \frac{d}{dr} E = 0$$

Its simplification leads to the equation **#4**

$$\#4: \quad \alpha \cdot m \cdot n \cdot r^{-n-1} - \frac{L}{m \cdot r^3} = 0$$

which we solve, let us say with respect to the parameter α

$$\#5: \quad \alpha = \frac{L^2 \cdot r^{n-2}}{m^2 \cdot n}$$

We rewrite #**5** in the form of definition **#6** (elimination of α) and try to solve the inequality (simplification of **#7**), ***DERIVE*** displays inequality **#8**:

$$\#6: \quad \left[\alpha := \frac{L^2 \cdot r^{n-2}}{m^2 \cdot n}\right]$$

$$\#7: \quad \text{SOLVE}\left(\left(\frac{d}{dr}\right)^2 E>0, n\right)$$

$$\#8: \quad [n \cdot (n-2) \cdot \text{SIGN}(m)>0]$$

However if we simplify **#7** but after the declaration **#9** *DERIVE* returns the solution **#10**

$$\#9: \quad m :\in \text{Real } (0, \infty)$$

$$\#10: \quad [n<0, n>2]$$

Comment:

We have started the solution procedure from the elimination of the parameter α. We could start also, for example, from the elimination of ***L*** :

$$\#1: \quad E := \frac{L^2}{2 \cdot m \cdot r^2} - \frac{\alpha \cdot m}{r^n}$$

$$\#2: \quad \text{SOLVE}\left(\frac{d}{dr} E=0, L\right) = \left[L = \frac{\sqrt{\alpha} \cdot m \cdot \sqrt{n} \cdot r}{\sqrt{(r^n)}}, L = -\frac{\sqrt{\alpha} \cdot m \cdot \sqrt{n} \cdot r}{\sqrt{(r^n)}}\right]$$

$$\#3: \quad L := \frac{\sqrt{\alpha} \cdot m \cdot \sqrt{n} \cdot r}{\sqrt{(r^n)}}$$

In this case instead of the solution we get the modified inequality **#6**:

```
#4:  [α:∈Real (0, ∞),m:∈Real (0, ∞),r:∈Real (0, ∞)]
```

$$\#5: \quad \mathrm{SOLVE}\left(\left(\frac{d}{dr}\right)^2 E>0, n\right)$$

$$\#6: \quad \left[n \cdot r^{-n-2} \cdot (n+1) \cdot (n-2) > 0\right]$$

The inequality #6 is to difficult for DERIVE. However it can be easily seen that its solution is $n > 2$ and $-1 < n < 0$ (see bellow)

```
#7:  SOLVE(n·(n+1)·(n-2)>0,n)

#8:  [n>2,-1<n<0]
```

The negative values of *n* correspond the minimum in the infinity.

Problem 5

The Fourier expansion is performed by the **FOURIER** function from the utility file DIF_APPS.MTH. Let us suppose that we try to evaluate Fourier coefficients. Below are the obtained integration results:

```
#1:  m:∈Integer [0, ∞)
```

$$\#2: \quad \int_{-\pi}^{\pi} \mathrm{COS}(m \cdot x) \cdot \mathrm{COS}(m \cdot x)\, dx$$

$$\#3: \quad \pi$$

(For m=0 the integral #2 is equal 2¶)

```
#4:  [n:∈Integer (0, ∞),m:∈Integer (0, ∞)]
```

$$\#5: \quad \int_{-\pi}^{\pi} \mathrm{COS}(n \cdot x) \cdot \mathrm{COS}(m \cdot x)\, dx$$

$$\#6: \quad 0$$

The correct result is ¶*KRONECKER(m.,n)

Example 6

Many physical problems can be solve in the relatively simple way with the use of the Lagrange equations of motion. For systems that conserve energy they have the form:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = 0,$$

where L is the Lagrangian, and $q_k, \dot{q}_k$ denote the k*th* generalized coordinate and the k*th* generalized velocity, respectively. The Lagrangian is the difference of kinetic and potential energy:

$$L = Ek - Ep,$$

If we solve Lagrange equations we should remember that ***DERIVE*** will not accept the differentiation variable $\dot{q}_k$. It means that ***DERIVE*** does not accept the notation

DIF(L, DIF(qk(t), t)).

To omit this inconvenience we enter the empty assignment for the generalized velocity as a function of time *t* and additionally we enter new variable for the generalized velocity. For example:

Qkp : = qkp(t) :=

DIF(DIF(L, Qkp), t) - DIF(L, qk) = 0

Letter **p** represents the dot sign above the variable.

Conclusion

Comparing various computer algebra systems (*Mathematica*, *Maple*, *Derive*, *MathCad*, *Reduce*) from the point of view of comfort of their use, system requirements, price, applicability to physics specific problems etc. one can conclude that in despite of the presented shortcomings ***DERIVE*** is a very satisfactory system, because of its efficiency and comfort of use, and low purchase price at the same time.