MAPLE AND THE ABSTRACTION PROCESS

Kiki Ariyanti Sugeng
kiki @makara.cso.ui.ac.id
Dept. of Mathematics – University of Indonesia
Depok 16424 , Indonesia

ABSTRACT

This paper is motivated by the difficulties of our students to make a graph of two variables functions. In many cases, a new concept and the process abstraction can be introduced using dynamical visualization. Most of our students will be able to generalizing a new concept from the examples.

The objection of this paper is to share experiences in teaching by using Maple. Maple can help student to understand the abstraction process. The students should know that the abstraction is important for mathematics development. They will learn about this with the help of technology. We must try very hard to implement visual mathematics curriculum.

1. INTRODUCTION

Mathematical Teaching Method used in our university is usually a classical method that use the following sequence:

Definition → Theorem → Proof → Examples → Test.

This method has been used for many years so that it is believed to be the standard method to teach mathematics. It also used with "paper and pencil " technique. So far, in my experience with mathematics teaching, especially in first year level, the students pay to much attention in computational process. This situation brings some weaknesses. Our students can not see the process of abstraction, whereas the abstraction process holds an important role in understanding and developing of mathematics. It has an impact to the advanced courses, such as Analysis and Algebra.

As we know, mathematics is different from other disciplines. It deals with abstraction process. It has its own symbolic language and it is perfect in the sense that new theories do not change it. But it is known that most of students have a problem to understand this abstraction process. The other weakness is our students also find difficulties whenever they must draw a graph of two variable function. Of course they can draw the simple function easily, but they find the difficulties to draw the complicated surface, such as z = cos (xy), z = $\frac{x-y}{x^2+y^2}$.

As another example, the concept of vector space is also difficult to understand. Usually the students don't have any difficulties to understand R^n spaces, but they can't understand the abstract space very well.

Using CAS such as MAPLE may solve this difficulty. With the help of CAS, the complicated computational processes can be reduced to enable students to focus more on the analysis of the problem. Moreover, students will be able to do a lot of experiments, especially with the visualization problems. In this case, we may spare them the usual boringness in understanding abstraction process, so in the long run they will be able to understand this process better.

II. THE BENEFIT OF USING MAPLE .

Working through experiences is the accepted method of learning mathematics. With Maple's help, student can do a lot of exercises than work with the classical way, Not only the number of exercises will increase but also the variety of the problem.

Chundang [3] discusses about the introduction of the concepts in calculus of several variables. He said that a deep understanding of the basic concepts are important in the further study. Having better understanding of basic concepts, the students will have a better understanding of the abstraction process. We know that in mathematics there are a lot of concepts that is an abstraction from another basic concepts. As an example, several variables functions is an abstraction of the one variable function, vector space is an abstract concept of the Real Space, Measure is an abstraction concepts of length of interval, e.t.c.

For the first example, we discuss the maximum/minimum problem in several variables functions. The students usually don't have any problem to find the maximum/minimum values in one variable functions. They can connect easily the concept of maximum/minimum and gradient of function. It is happen because they can draw the functions easily without a help of computer technology. But the problem appears whenever they must solve the maximum/minimum problem in two variables function. The problems appear whenever they must draw the two variables functions. The teachers are not always good in drawing. So they only give a simple function, such as $z = x^2 + y^2$. This simple example will make the students can not have a good understanding that the several variables function, like the one variables functions, also can have several extreme points or even no extreme point. The students also can not see the connection between the gradient and the maximum/minimum problem. This problem will be overcome with Maple's help. Maple has a good visualization capability. So the students will overcome their problems in drawing functions easily. They can find the surface of various functions, such as $z = \dfrac{x-y}{x^2+y^2}$, Z= cos (xy) and z= $-\dfrac{x^2}{4}+\dfrac{y^2}{9}$. From these three examples, the students can see which function has a unique maximum and minimum, many maximum and minimum or no maximum nor minimum. From these examples, they also have a capability to connect the concept of gradient and the extreme point easily.

As we know that in text book there is written:

A necessary condition that z= f (x, y) have a relative maximum/minimum is

$$\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$$

It means that $\nabla f = \dfrac{\partial f}{\partial x} i + \dfrac{\partial f}{\partial y} j = 0$ or the tangent plane is parallel with the xy-plane.

If the student understand the above concept then it is hoped that the student will not have the difficulties if they must make the process of abstraction of this concept to several variables that can not be drawn anymore.

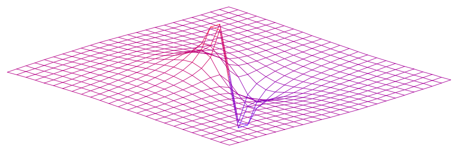

Figure 1. $z = \dfrac{x-y}{x^2+y^2}$.

This function has a unique maximum/minimum point.

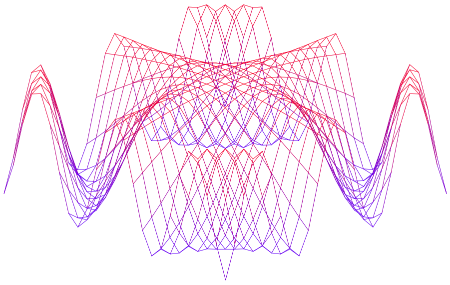

Figure 2. z=cos(xy).
This function has many relative maximum/minimum points.

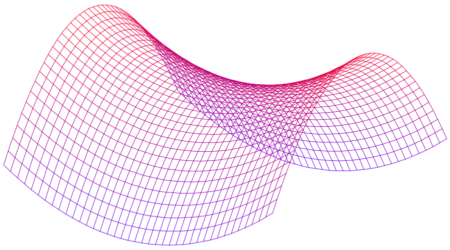

Figure 3. $z = -\frac{x^2}{4} + \frac{y^2}{9}$.
This function has no maximum/minimum point, but it has a saddle point.

The another example is about vector space in elementary linear algebra. It is difficult to explain and to make the students understand clearly about this abstract concept. Usually we give R^n space for the examples, but they still couldn't understand clearly about another space. As an example, if we talk about the M_{mxn} spaces. Maple will help the students to have a better understanding, because we can do the trial of the axioms of vector space with "randmatrix" help. In the following example, we want to check the close property of sum of 3x3-matrix and scalar multiplication. We can run the randmatrix several times to make sure that this property can be a hypothesis that the sum of matrix and the scalar multiplication have close property. After the students have the hypothesis then they can prove the concept theoretically.

```
>with (linalg):
> A:=randmatrix(3,3);
                    [-99    -85    -86]
                    [                 ]
               A := [ 30     80     72 ]
                    [                 ]
                    [ 66    -29    -91]
```

```
> B:=randmatrix(3,3);

                  [ 19   -50   88]

             B := [-53    85   49]
                  [              ]
                  [ 78    17   72]

> A+B=matadd(A,B);

                   [-39   -140   141]
                   [                ]
             A + B = [-54    179   132]
                   [                ]
                   [ -8     40    -12]

> k:=rand();

               k := 912854227507

> scalarmul(A,k);

    [-90372568523193   -77592609338095   -78505463565602]
    [                                                   ]
    [ 27385626825210    73028338200560    65725504380504 ]
    [                                                   ]
    [ 60248379015462   -26472772597703   -83069734703137]
```

We can try another maple's functions to have another properties. So with Maple's help, the student will have enough time to understand the analysis and abstraction process better, because the computational and the visualization works have taken over by Maple

III. CONCLUSION.

Technology changes the role of the teacher and the method of mathematics teaching. Technology also changes attitude of the student, they enjoy and feel enthusiastic for study mathematics with exploring and creating method. But our teaching methods are still so conservative and very slow to respond the changing circumstances. However the world is changing so fast. Technology must be used to developing our teaching method. Many of us would say it is mainly due to the lack of fund. But actually that is not the really problem. In my opinion, awareness and enthusiasm to change are more important than material or finance condition itself. In my first time using Maple, we only have 10 PC for 60 students. But we can manage the class, the limited number of PC could serve the students in 3 shifts lab. hours. Thank God, at the end of this year we will have around 60 PC and we will not have a problem to serve the students anymore.

Using Maple or CAS , we will have a new teaching method. This new teaching method has The following sequences :

Problem → explore → hypothesis → proof → theorem.

Of course we can mix this method with the classical method to having better result.

Ruth Hubbard [2] said that there are three basic kinds of problem-based courses that we could use as a model to design our courses:

1. Mathematical modeling courses which dealing with real world problem.
2. Courses in solving abstract mathematical problems.
3. Courses which combine (1) and (2) with computer technology.

With the above new sequence, we can implement the problem-based courses better. Students also feel easier to understand the abstraction process, because they only take a few times to make computations and they can have more attention for the analysis process. So we can realize visual mathematics courses, but we must work very hard . If we do the work together , there is no big problem. But the most important thing is the willingness to change the view and the attitude of teaching.

IV. REFERENCES.

1. U. Chundang , Using CAS for the visualization of some basic concepts in calculus of several variables, TCM Conference, Japan , 1998.
2. R. Hubbard , 53 Interesting Way to Teach Mathematics , TES pub. , 1990.
3. Purcell,E.J. & Varberg, D. , Calculus with Analytic Geometry , Prentice Hall, 1997.